# Norms and Standardized Scores

The following inputs raw scores from Table 2.1 of the text.

```
# dput({set.seed(1421); sample(rep(25:36, c(3, 8, 7, 5, 5, 7, 3, 2, 2, 6, 2, 2)))})
raw_score <- c(26, 25, 33, 31, 26, 34, 29, 36, 25, 29, 28, 32, 25,
               30, 27, 31, 30, 30, 35, 30, 27, 26, 34, 32, 26, 34,
               30, 28, 28, 31, 30, 27, 26, 29, 29, 33, 27, 35, 26,
               27, 28, 29, 28, 27, 34, 36, 26, 26, 34, 30, 34, 27)
```

### Frequency Distribution

The following recreates Table 2.1 using R.

```
freq <- table(raw_score)  # frequency
cumfreq <- cumsum(freq)  # cumulative frequency
perc <- prop.table(freq) * 100  # percentage
cumperc <- cumsum(perc)  # cumulative percentage
pr <- (cumperc - 0.5 * perc)  # percentile rank
cbind(freq, cumfreq, perc, cumperc, pr)
```

```
   freq cumfreq      perc    cumperc        pr
25    3       3  5.769231   5.769231  2.884615
26    8      11 15.384615  21.153846 13.461538
27    7      18 13.461538  34.615385 27.884615
28    5      23  9.615385  44.230769 39.423077
29    5      28  9.615385  53.846154 49.038462
30    7      35 13.461538  67.307692 60.576923
31    3      38  5.769231  73.076923 70.192308
32    2      40  3.846154  76.923077 75.000000
33    2      42  3.846154  80.769231 78.846154
34    6      48 11.538462  92.307692 86.538462
35    2      50  3.846154  96.153846 94.230769
36    2      52  3.846154 100.000000 98.076923
```

### Percentile Points

Score points for a particular percentile ranks

```
# P74
quantile(raw_score, .74)
```

```
  74%
31.74
```

```
# Use a different type (see https://en.wikipedia.org/wiki/Quantile#Estimating_quantiles_from_
quantile(raw_score, .74, type = 6)
```

```
74%
 32
```

### Standardized Scores

$z$-score

```
z_score <- (raw_score - mean(raw_score)) / sd(raw_score)
c(mean = mean(z_score), sd = sd(z_score))
```

```
         mean            sd
-4.937123e-16  1.000000e+00
```

$T$-score

```
T_score <- z_score * 10 + 50
c(mean = mean(T_score), sd = sd(T_score))
```

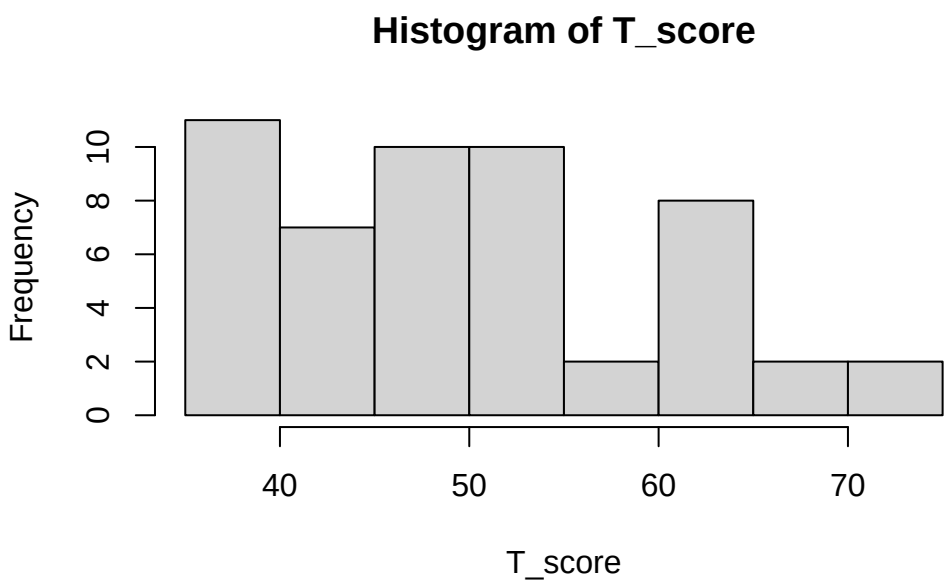
```
mean   sd
  50   10
```

> **Important**
>
> Standardization does not change the shape of the distribution.

```r
hist(raw_score)
```


```r
hist(z_score)
```

```
hist(T_score)
```

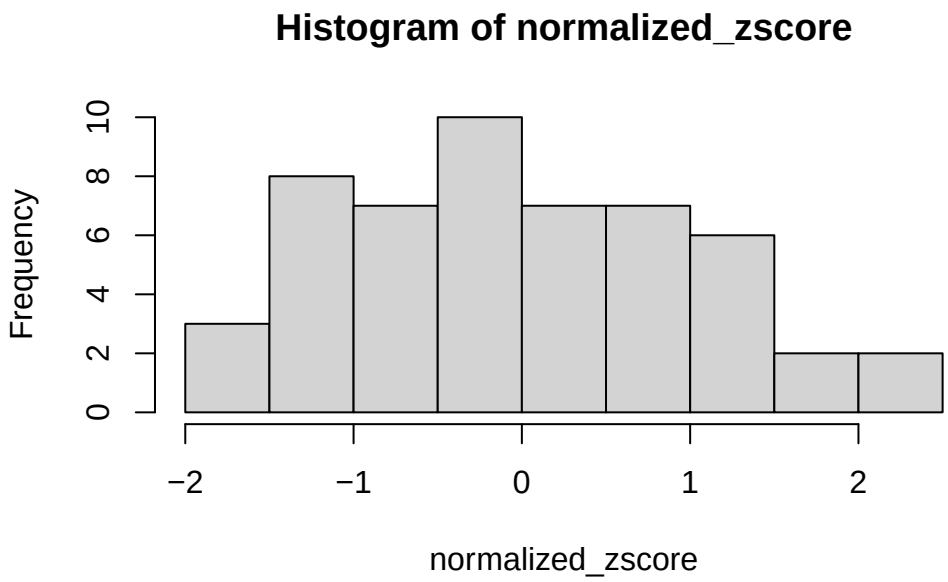



However, the histograms may look slightly different as each plot uses somewhat different ways to bin the values. But if you compute the skewness and the kurtosis, they should not be affected by the transformation.

### Normalized Scores

Normalized $z$-score

```
# Using normal quantile
qnorm_pr <- qnorm(pr / 100)
# Convert raw scores
normalized_zscore <- as.vector(qnorm_pr[as.character(raw_score)])
hist(normalized_zscore)  # the shape will be closer to normal
```




### Age/Grade Equivalents

The following shows some fake data with two groups, one at age 5 and the other at age 6. The two groups took different forms of the test, and each form has 12 anchor/common items and 24 other items. The total score is the sum of the anchor and other items.

```
# Create data
y5 <- structure(c(14L, 12L, 18L, 8L, 12L, 20L, 16L, 18L, 11L, 21L,
11L, 20L, 18L, 14L, 20L, 13L, 11L, 17L, 19L, 16L, 18L, 14L, 14L,
19L, 18L, 4L, 4L, 12L, 8L, 28L, 18L, 12L, 16L, 9L, 10L, 18L,
12L, 12L, 21L, 15L, 22L, 12L, 8L, 11L, 18L, 10L, 14L, 14L, 5L,
8L, 11L, 16L, 11L, 13L, 10L, 12L, 8L, 18L, 18L, 15L, 17L, 19L,
21L, 15L, 22L, 12L, 15L, 15L, 22L, 20L, 11L, 15L, 16L, 13L, 17L,
17L, 19L, 11L, 13L, 15L, 15L, 12L, 16L, 12L, 15L, 16L, 18L, 15L,
21L, 21L, 18L, 7L, 15L, 18L, 18L, 16L, 16L, 18L, 17L, 19L, 5L,
2L, 8L, 2L, 2L, 4L, 3L, 7L, 5L, 7L, 6L, 6L, 6L, 4L, 8L, 3L, 3L,
3L, 2L, 6L, 4L, 4L, 4L, 5L, 7L, 1L, 1L, 3L, 5L, 8L, 5L, 3L, 5L,
3L, 4L, 6L, 4L, 4L, 5L, 4L, 6L, 2L, 4L, 2L, 3L, 6L, 4L, 3L, 0L,
1L, 3L, 5L, 3L, 6L, 2L, 1L, 2L, 5L, 7L, 6L, 5L, 5L, 8L, 3L, 5L,
3L, 3L, 3L, 6L, 4L, 5L, 5L, 4L, 4L, 6L, 2L, 7L, 3L, 6L, 4L, 4L,
4L, 6L, 3L, 6L, 6L, 7L, 3L, 6L, 6L, 4L, 0L, 4L, 5L, 4L, 6L, 4L,
5L, 6L, 6L), dim = c(100L, 2L), dimnames = list(NULL, c("total",
```

```
"anchor")))
y6 <- structure(c(20L, 15L, 10L, 12L, 20L, 14L, 13L, 15L, 24L, 22L,
15L, 16L, 21L, 24L, 9L, 19L, 12L, 13L, 14L, 18L, 10L, 23L, 19L,
11L, 12L, 13L, 18L, 21L, 22L, 22L, 24L, 16L, 13L, 4L, 17L, 24L,
12L, 14L, 23L, 12L, 20L, 16L, 20L, 15L, 12L, 20L, 14L, 8L, 24L,
5L, 26L, 23L, 12L, 15L, 18L, 19L, 18L, 11L, 16L, 17L, 18L, 16L,
19L, 18L, 17L, 17L, 20L, 10L, 12L, 14L, 18L, 10L, 16L, 21L, 15L,
14L, 9L, 13L, 18L, 15L, 4L, 19L, 16L, 21L, 14L, 15L, 26L, 23L,
21L, 20L, 17L, 13L, 10L, 15L, 13L, 21L, 17L, 18L, 24L, 18L, 5L,
5L, 3L, 6L, 7L, 6L, 4L, 5L, 5L, 8L, 4L, 7L, 8L, 7L, 5L, 8L, 3L,
4L, 5L, 6L, 3L, 9L, 6L, 5L, 4L, 3L, 5L, 7L, 8L, 8L, 6L, 5L, 6L,
2L, 5L, 6L, 4L, 6L, 8L, 3L, 7L, 4L, 6L, 5L, 5L, 9L, 7L, 2L, 9L,
2L, 9L, 8L, 3L, 5L, 8L, 6L, 7L, 3L, 6L, 6L, 5L, 6L, 8L, 6L, 8L,
6L, 10L, 6L, 2L, 6L, 6L, 5L, 8L, 8L, 8L, 8L, 3L, 5L, 7L, 5L,
0L, 7L, 7L, 9L, 4L, 5L, 6L, 7L, 6L, 7L, 8L, 6L, 1L, 2L, 5L, 6L,
6L, 7L, 10L, 6L), dim = c(100L, 2L), dimnames = list(NULL, c("total",
"anchor")))
```

**Questions**

### Q1

Compute the means of total scores for the two groups. Do the two groups show differences in their total scores?

```
# Your code
```

### Q2

Repeat Q1 for only the anchor items.

```
# Your code
```

### Q3

Repeat Q1 for only the non-anchor items.

```
# Your code
```

### Q4

Do you think the difficulty levels of the non-anchor items are the same for the test forms? Explain your reasoning.

### Q5

The median of the 5-year-old group is 15, so someone taking form A (the form for the 5-year-old group) with a score of 15 will have an AE (age-equivalent score) of 5.
What is the issue to say that someone taking form B (the form for the 6-year-old group) with a score of 15 will have an AE of 5?

### Q6

What do you think should be the score on form B that corresponds to an AE of 5? (There is no one right answer here, and you should explain your logic.)

```
# Type your code if needed
```