Exploratory Factor Analysis

```r
library(tidyverse)
library(psych)
library(dagitty)
library(plotly)
library(lavaan)
library(EFAtools)
library(flextable)
```

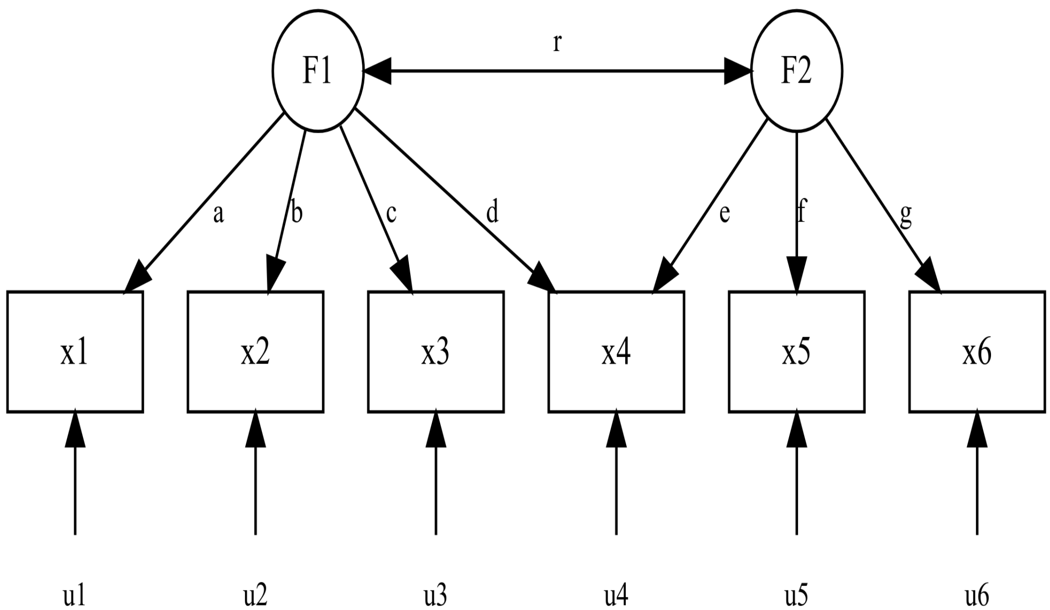
Path Diagram

Personally, it may be easier to use powerpoint or other tools with a graphical interface. But if you want to do it with code, here is an example with Graphviz.

Graphviz

```
digraph FactorModel {
    node [shape=circle,fontsize=12];
    F1; F2;

    node [shape=rectangle];
    x1; x2; x3; x4; x5; x6;

    edge [fontsize=10];
    F2 -> F1 [label="r" constraint=false dir="both"];
    F1 -> x1 [label="a"];
    F1 -> x2 [label="b"];
    F1 -> x3 [label="c"];
    F1 -> x4 [label="d"]
```

```
    F2 -> x4 [label="e"];
    F2 -> x5 [label="f"];
    F2 -> x6 [label="g"];

    node [shape=none,fontsize=10];
    u1 -> x1;
    u2 -> x2;
    u3 -> x3;
    u4 -> x4;
    u5 -> x5;
    u6 -> x6;

    {rank=same; x1 x2 x3 x4 x5 x6}
    {rank=min; F1 F2}
    {rank=max; u1 u2 u3 u4 u5 u6}
}
```

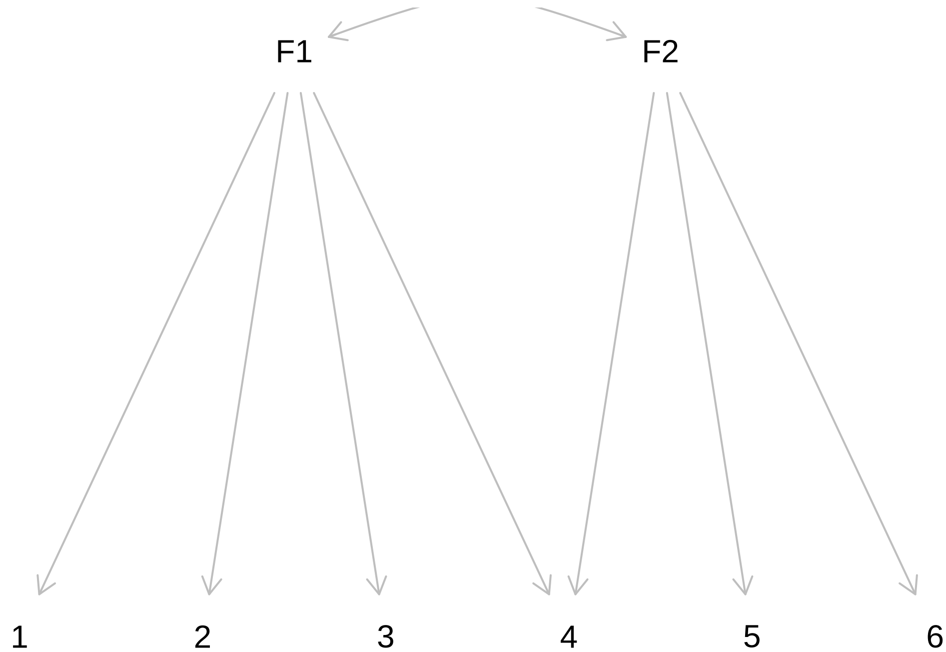


Path diagram of a two-factor model.

DAG

A factor model can also be represented as a directed acyclic graph (DAG). The `dagitty` package can show the vanishing tetrad assumptions:

```
dag1 <- dagitty("dag{ F1 -> {1 2 3 4}; F2 -> {4 5 6}; F1 <-> F2 }")
latents(dag1) <- c("F1", "F2")
coordinates(dag1) <- list(
    x = c(F1 = 0, F2 = 2, `1` = -1.5, `2` = -0.5, `3` = 0.5, `4` = 1.5,
          `5` = 2.5, `6` = 3.5),
    y = c(F1 = 0, F2 = 0, `1` = 1, `2` = 1, `3` = 1, `4` = 1,
          `5` = 1, `6` = 1)
)
plot(dag1)
```




```
vanishingTetrads(dag1)
```

```
     [,1] [,2] [,3] [,4]
[1,] "1"  "3"  "4"  "2"
[2,] "1"  "2"  "4"  "3"
[3,] "1"  "2"  "3"  "4"
[4,] "1"  "3"  "5"  "2"
```

```
 [5,] "1"  "2"  "5"  "3"
 [6,] "1"  "2"  "3"  "5"
 [7,] "1"  "3"  "6"  "2"
 [8,] "1"  "2"  "6"  "3"
 [9,] "1"  "2"  "3"  "6"
[10,] "1"  "4"  "5"  "2"
[11,] "1"  "4"  "6"  "2"
[12,] "1"  "5"  "6"  "2"
[13,] "1"  "4"  "5"  "3"
[14,] "1"  "4"  "6"  "3"
[15,] "1"  "5"  "6"  "3"
[16,] "1"  "5"  "6"  "4"
[17,] "2"  "4"  "5"  "3"
[18,] "2"  "4"  "6"  "3"
[19,] "2"  "5"  "6"  "3"
[20,] "2"  "5"  "6"  "4"
[21,] "3"  "5"  "6"  "4"
```

Correlation Matrix

```
# Covariance
cov_oa <- cov(
    select(bfi, O1:O5, A1:A5),
    # Listwise deletion
    use = "complete"
)
# Correlation
corr_oa <- cor(
    select(bfi, O1:O5, A1:A5),
    # Listwise deletion
    use = "complete"
)
round(corr_oa, digits = 2)
```

```
      O1    O2    O3    O4    O5    A1    A2    A3    A4    A5
O1  1.00 -0.22  0.39  0.19 -0.24  0.01  0.13  0.15  0.06  0.16
O2 -0.22  1.00 -0.28 -0.08  0.33  0.07  0.01  0.00  0.04 -0.01
O3  0.39 -0.28  1.00  0.19 -0.32 -0.06  0.16  0.23  0.07  0.24
O4  0.19 -0.08  0.19  1.00 -0.18 -0.08  0.08  0.04 -0.04  0.02
O5 -0.24  0.33 -0.32 -0.18  1.00  0.11 -0.09 -0.06  0.02 -0.06
```

```
A1  0.01  0.07 -0.06 -0.08  0.11  1.00 -0.34 -0.27 -0.15 -0.19
A2  0.13  0.01  0.16  0.08 -0.09 -0.34  1.00  0.49  0.33  0.39
A3  0.15  0.00  0.23  0.04 -0.06 -0.27  0.49  1.00  0.37  0.51
A4  0.06  0.04  0.07 -0.04  0.02 -0.15  0.33  0.37  1.00  0.31
A5  0.16 -0.01  0.24  0.02 -0.06 -0.19  0.39  0.51  0.31  1.00
```

Polychoric Correlation

The items are technically ordinal, but with six points it is usually less a problem to just use Pearson correlation. We can use the `lavaan` package to get the polychoric correlation.

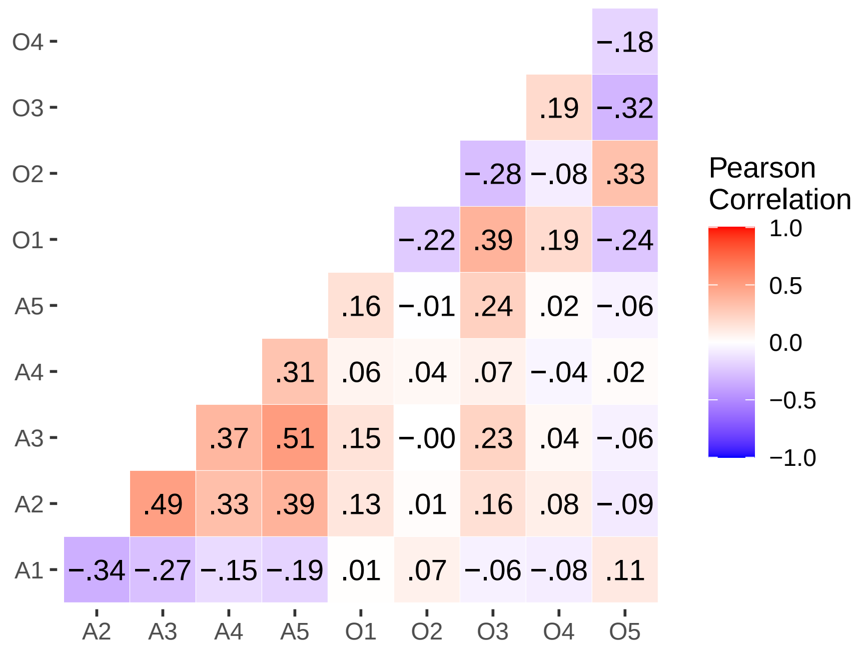
```
pcorr_oa <- lavaan::lavCor(select(bfi, O1:O5, A1:A5), ordered = TRUE)
pcorr_oa
```

```
       O1     O2     O3     O4     O5     A1     A2     A3     A4     A5
O1  1.000
O2 -0.273  1.000
O3  0.450 -0.337  1.000
O4  0.258 -0.106  0.255  1.000
O5 -0.304  0.373 -0.381 -0.250  1.000
A1 -0.010  0.079 -0.086 -0.100  0.134  1.000
A2  0.163 -0.002  0.189  0.109 -0.114 -0.411  1.000
A3  0.187 -0.029  0.270  0.065 -0.089 -0.327  0.565  1.000
A4  0.071  0.035  0.082 -0.049  0.026 -0.182  0.390  0.422  1.000
A5  0.195 -0.018  0.275  0.028 -0.074 -0.234  0.453  0.576  0.358  1.000
```

Heat Map

```
corr_oa |>
    as_tibble(rownames = "Var1") |>
    pivot_longer(cols = -Var1, names_to = "Var2") |>
    mutate(across(1:2, .fns = as.ordered)) |>
    filter(Var2 > Var1) |>
    ggplot(aes(x = Var2, y = Var1, fill = value)) +
    geom_tile(color = "white") +
    geom_text(aes(Var2, Var1, label = rmlead0(value)), color = "black", size = 4) +
    scale_fill_gradient2(low = "blue", high = "red", limit = c(-1, 1), name = "Pearson\nCorre
    theme(axis.title.x = element_blank(),
          axis.title.y = element_blank(),
```

```
        panel.background = element_blank()) +
  coord_fixed()
```



Factor Extraction

Eigen-Decomposition

A good video to explain this: https://www.youtube.com/watch?v=86ODrk1nB-g

```
# Simulate 3-D Data
lambda_mat <- matrix(c(.8, .7, .5, .1, .5, .4), nrow = 3)
sigma_mat <- tcrossprod(lambda_mat)
diag(sigma_mat) <- 1
y <- MASS::mvrnorm(500, mu = rep(0, 3), Sigma = sigma_mat, empirical = TRUE)
```

```
# library(scatterplot3d)
# scatterplot3d(y)
dfy <- as.data.frame(y)
names(dfy) <- c("y1", "y2", "y3")
plot_ly(dfy, x = ~ y1, y = ~ y2, z = ~ y3)
```

```
No trace type specified:
  Based on info supplied, a 'scatter3d' trace seems appropriate.
  Read more about this trace type -> https://plotly.com/r/reference/#scatter3d

No scatter3d mode specifed:
  Setting the mode to markers
  Read more about this attribute -> https://plotly.com/r/reference/#scatter-mode

file:////tmp/RtmpKvOBq0/file32987171e6e980/widget3298716a1d761b.html screenshot completed
```

Estimating Communalities

Squared multiple correlations (R^2)

Use squared multiple R^2 (not efficient)

```r
# E.g., Regress A1 on all other variables
summary(lm(A1 ~ ., data = select(bfi, O1:O5, A1:A5)))
```

```
Call:
lm(formula = A1 ~ ., data = select(bfi, O1:O5, A1:A5))

Residuals:
    Min      1Q  Median      3Q     Max 
-3.5113 -0.9180 -0.3207  0.7330  4.5763 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.95198    0.20993  18.825  < 2e-16 ***
O1           0.12114    0.02488   4.869 1.19e-06 ***
O2           0.07446    0.01761   4.228 2.44e-05 ***
O3           0.04967    0.02429   2.045  0.04098 *  
O4          -0.06568    0.02182  -3.010  0.00263 ** 
O5           0.08494    0.02114   4.018 6.03e-05 ***
A2          -0.32318    0.02565 -12.599  < 2e-16 ***
A3          -0.13979    0.02469  -5.662 1.66e-08 ***
A4          -0.02005    0.01885  -1.063  0.28765    
A5          -0.03240    0.02404  -1.348  0.17780    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.295 on 2637 degrees of freedom
  (153 observations deleted due to missingness)
Multiple R-squared:  0.1541,    Adjusted R-squared:  0.1512 
F-statistic: 53.37 on 9 and 2637 DF,  p-value: < 2.2e-16
```

```r
# E.g., Regress O1 on all other variables
summary(lm(O1 ~ ., data = select(bfi, O1:O5, A1:A5)))
```

```
Call:
lm(formula = O1 ~ ., data = select(bfi, O1:O5, A1:A5))

Residuals:
    Min      1Q  Median      3Q     Max
-4.5570 -0.5727  0.1064  0.7152  2.6925

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.730613   0.165923  16.457  < 2e-16 ***
O2          -0.080779   0.013678  -5.906 3.96e-09 ***
O3           0.264794   0.018230  14.525  < 2e-16 ***
O4           0.098933   0.016922   5.846 5.64e-09 ***
O5          -0.082739   0.016445  -5.031 5.20e-07 ***
A1           0.073559   0.015107   4.869 1.19e-06 ***
A2           0.052696   0.020556   2.564  0.01042 *
A3           0.032197   0.019345   1.664  0.09616 .
A4           0.008584   0.014692   0.584  0.55911
A5           0.049964   0.018712   2.670  0.00763 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.009 on 2637 degrees of freedom
  (153 observations deleted due to missingness)
Multiple R-squared:  0.2031,    Adjusted R-squared:  0.2004
F-statistic: 74.67 on 9 and 2637 DF,  p-value: < 2.2e-16
```

A computational short cut is to use the inverse of the correlation matrix

```r
1 - 1 / diag(solve(corr_oa))
```

```
       O1        O2        O3        O4        O5        A1        A2        A3
0.2030977 0.1625871 0.2766737 0.0766234 0.1910893 0.1540951 0.3401059 0.4002928
       A4        A5
0.1885574 0.3138769
```

Iterative method

Here is sample code for principal axis factoring (using `eigen`)

```r
rcorr_oa <- corr_oa # initialize reduced correlation
diag(rcorr_oa) <- 1 - 1 / diag(solve(corr_oa))  # replace communality
q <- 2  # number of factors
eps <- .0001  # convergence criterion
while (TRUE) {
    eigen_r <- eigen(rcorr_oa, symmetric = TRUE)
    lambdat <- t(eigen_r$vectors[, seq_len(q)]) * sqrt(eigen_r$values[seq_len(q)])
    # communality = sum of squared loadings
    new_h <- colSums(lambdat^2)
    if (max(abs(new_h - diag(rcorr_oa))) > eps) {
        diag(rcorr_oa) <- new_h
    } else {
        break  # stop when convergence is met
    }
}
# Estimates with the iterative method
diag(rcorr_oa)
```

```
        O1         O2         O3         O4         O5         A1         A2
0.28265008 0.23396855 0.43541770 0.08877585 0.29766819 0.13249392 0.45364581
        A3         A4         A5
0.58815163 0.25098540 0.39616917
```

The final estimates are different from the initial ones.

Loading Matrix

```r
# From our for loop for PAF
t(lambdat)
```

```
           [,1]       [,2]
 [1,]  0.3548181 -0.3959189
 [2,] -0.1755868  0.4506955
 [3,]  0.4741739 -0.4589373
 [4,]  0.1651205 -0.2480128
 [5,] -0.2778184  0.4695370
 [6,] -0.3486927 -0.1044201
 [7,]  0.6303503  0.2372005
 [8,]  0.7164531  0.2737558
```

```
 [9,]  0.4173173  0.2771697
[10,]  0.6018286  0.1842694
```

```r
# Compared to `psych::fa()`
fa_unrotated <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2,
    rotate = "none",
    fm = "pa")
fa_unrotated$loadings
```

```
Loadings:
   PA1    PA2
O1  0.356 -0.392
O2 -0.166  0.442
O3  0.472 -0.453
O4  0.163 -0.244
O5 -0.270  0.476
A1 -0.345
A2  0.631  0.231
A3  0.705  0.271
A4  0.415  0.270
A5  0.601  0.187

                 PA1   PA2
SS loadings    2.025 1.084
Proportion Var 0.202 0.108
Cumulative Var 0.202 0.311
```

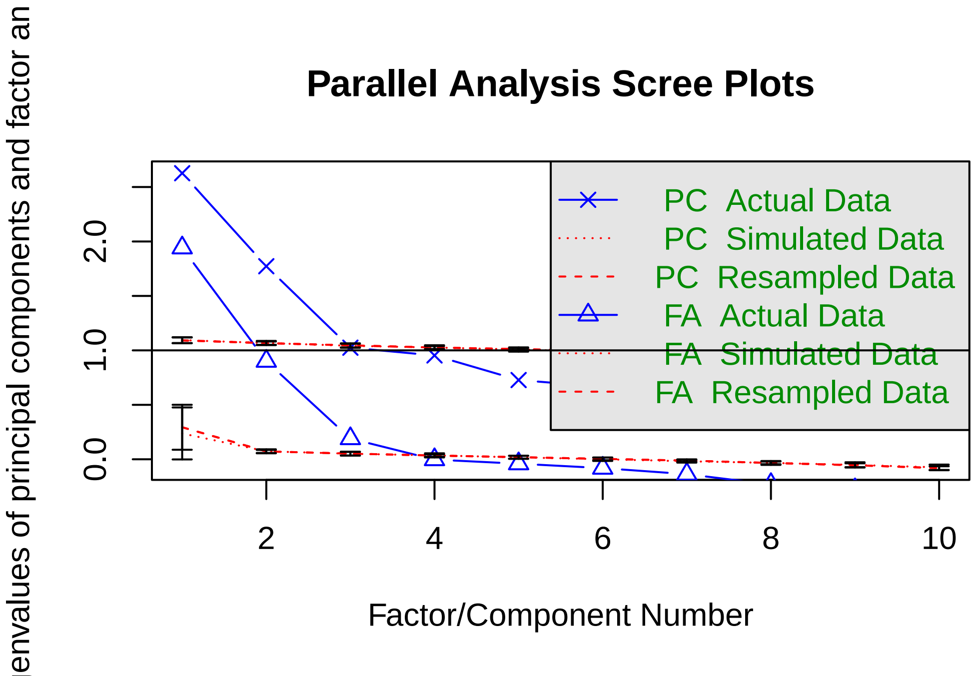
The values are not exactly the same as there are differences in implementation algorithms, but they are pretty close.

Number of Factors

Parallel Analysis

```r
fa.parallel(
    select(bfi, O1:O5, A1:A5),
```

```
    fm = "pa",
    error.bars = TRUE
)
```




```
Parallel analysis suggests that the number of factors =  3  and the number of components =  2
```

It suggests 3 factors (with reduced correlation) and 2 components (with full correlation). In my experience, the number of components is usually more in line with theoretical expectations, but choosing either one would seem reasonable, and this should depend on theoretical considerations and interpretability of the solutions.

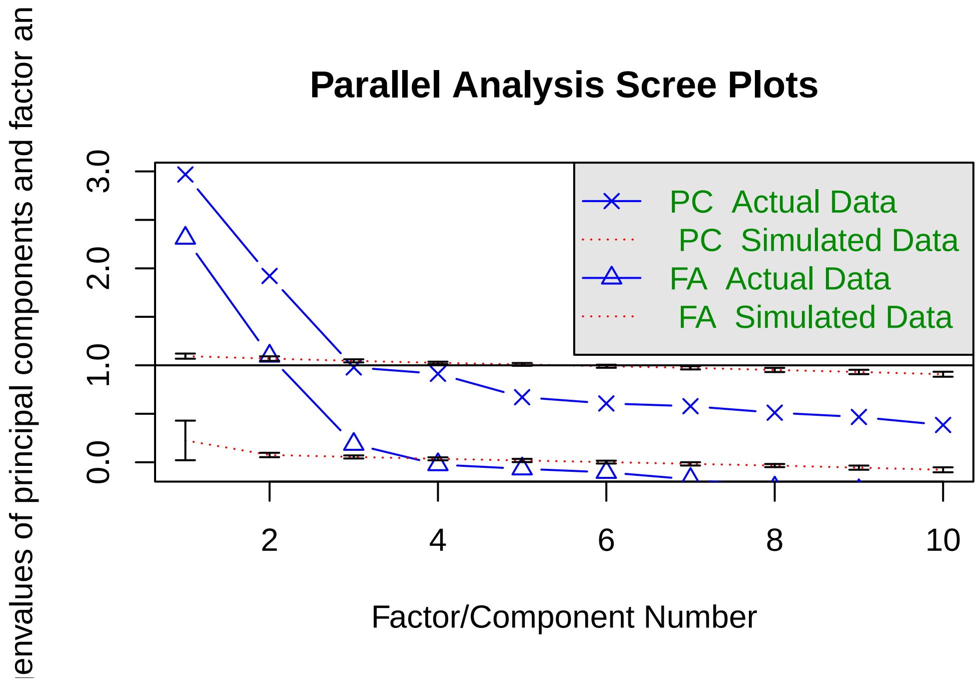
With polychoric correlation matrix

We need to first find the number of observations:

```
n_pcorr <-
    select(bfi, O1:O5, A1:A5) |>
    drop_na() |>
    nrow()
n_pcorr
```

```
[1] 2647
```

```r
psych::fa.parallel(
    pcorr_oa,
    n.obs = n_pcorr,
    fm = "pa",
    error.bars = TRUE
)
```




```
Parallel analysis suggests that the number of factors =  3  and the number of components =
```

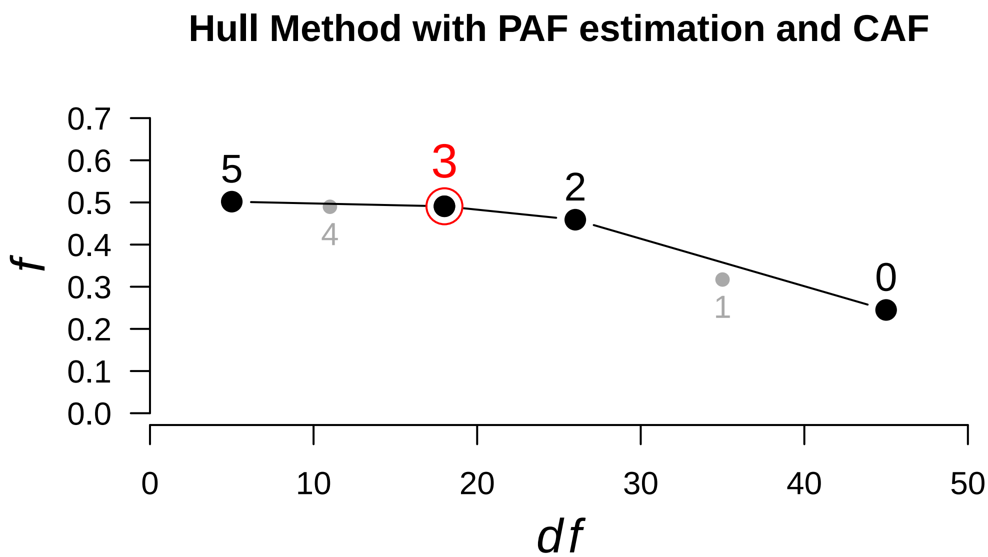
Hull method

```r
# Reduced correlation
HULL(select(bfi, O1:O5, A1:A5), eigen_type = "SMC")
```

```
i Only CAF can be used as gof if method "PAF" is used. Setting gof to "CAF"
```

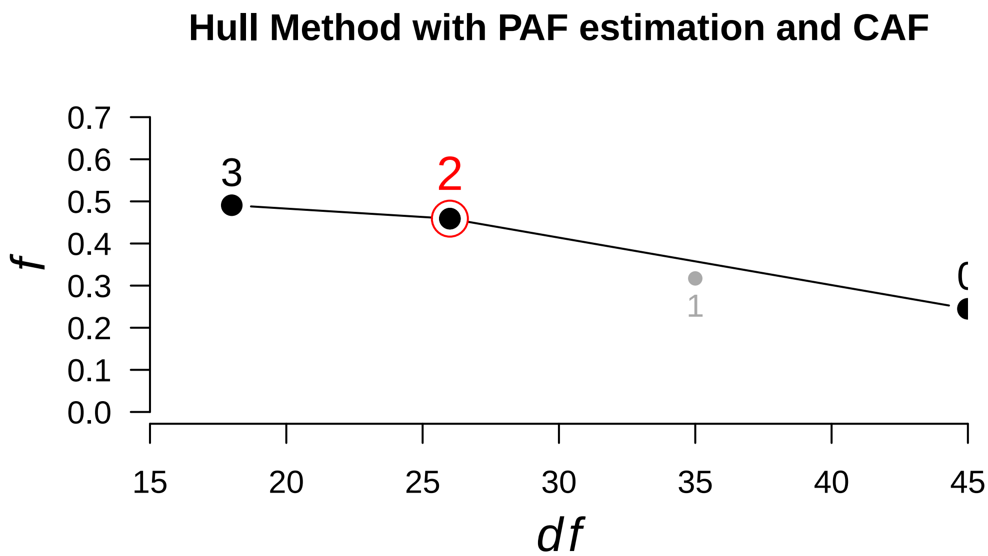
```
i 'x' was not a correlation matrix. Correlations are found from entered raw data.
```

```
Hull Analysis performed testing 0 to 5 factors.
PAF estimation and the CAF fit index was used.

-- Number of factors suggested by the Hull method -----------------------------

( ) With CAF:    3
```




```r
# Full correlation
HULL(select(bfi, O1:O5, A1:A5), eigen_type = "PCA")
```

```
i Only CAF can be used as gof if method "PAF" is used. Setting gof to "CAF"
i 'x' was not a correlation matrix. Correlations are found from entered raw data.


Hull Analysis performed testing 0 to 3 factors.
PAF estimation and the CAF fit index was used.

-- Number of factors suggested by the Hull method -----------------------------

( ) With CAF:    2
```

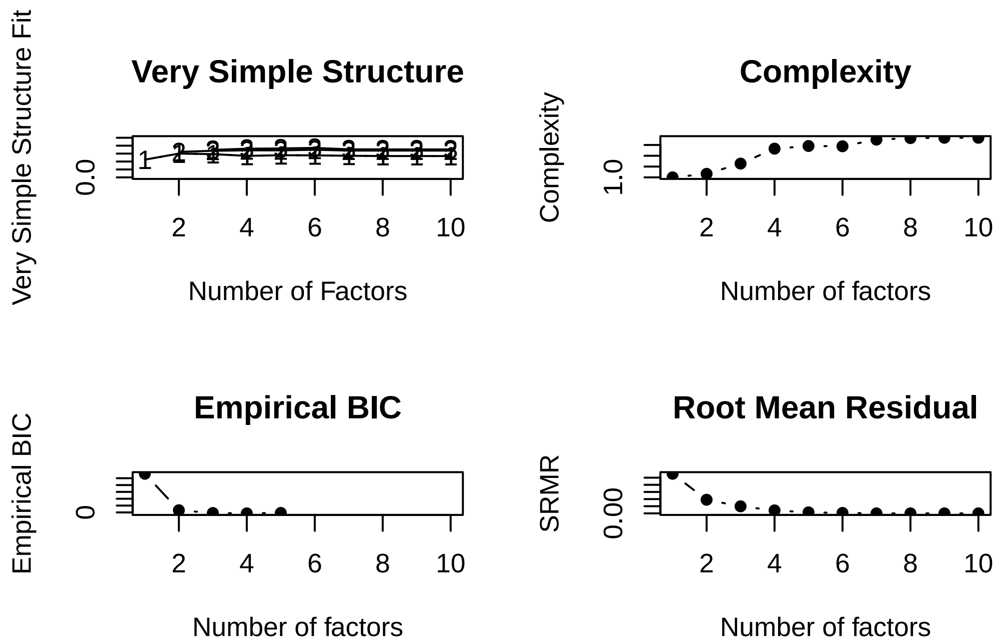


The conclusion is similar to parallel analysis.

Other methods

These should be examined as well. Note in this example, using `fm = "pa"` results in an error, so the suggestions are not based on PAF extraction.

```
psych::nfactors(select(bfi, O1:O5, A1:A5))
```




```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of 0.6  with  2  factors
VSS complexity 2 achieves a maximimum of 0.7  with  6  factors
The Velicer MAP achieves a minimum of 0.03  with  2  factors
Empirical BIC achieves a minimum of  -69.58  with  4  factors
Sample Size adjusted BIC achieves a minimum of  -27.76  with  4  factors

Statistics by number of factors
   vss1 vss2   map dof   chisq     prob sqresid  fit RMSEA  BIC  SABIC complex
1  0.45 0.00 0.036  35 1.6e+03 1.3e-311     7.9 0.45 0.126 1310 1421.4     1.0
2  0.60 0.64 0.032  26 3.2e+02  5.4e-53     5.1 0.64 0.064  117  199.1     1.1
3  0.59 0.67 0.052  18 9.5e+01  1.7e-12     4.3 0.69 0.039  -48    9.4     1.3
4  0.54 0.68 0.087  11 2.5e+01  1.0e-02     3.9 0.73 0.021  -63  -27.8     1.5
5  0.56 0.68 0.134   5 2.8e+00  7.2e-01     3.7 0.74 0.000  -37  -21.0     1.6
6  0.56 0.70 0.213   0 5.5e-01       NA     3.4 0.76    NA   NA     NA     1.6
7  0.55 0.68 0.314  -4 1.6e-07       NA     3.5 0.76    NA   NA     NA     1.7
8  0.54 0.68 0.650  -7 2.2e-07       NA     3.5 0.75    NA   NA     NA     1.7
9  0.54 0.68 1.000  -9 0.0e+00       NA     3.5 0.75    NA   NA     NA     1.7
10 0.54 0.68    NA -10 0.0e+00       NA     3.5 0.75    NA   NA     NA     1.7
    eChisq    SRMR  eCRMS eBIC
```

```
1  3.1e+03 1.1e-01 0.1259 2827
2  3.7e+02 3.8e-02 0.0502  160
3  9.8e+01 2.0e-02 0.0312  -45
4  1.8e+01 8.4e-03 0.0170  -70
5  2.1e+00 2.9e-03 0.0086  -38
6  3.7e-01 1.2e-03     NA   NA
7  1.2e-07 6.8e-07     NA   NA
8  1.4e-07 7.5e-07     NA   NA
9  1.1e-14 2.1e-10     NA   NA
10 1.1e-14 2.1e-10     NA   NA
```

Based on theoretical consideration, two factors seem reasonable.

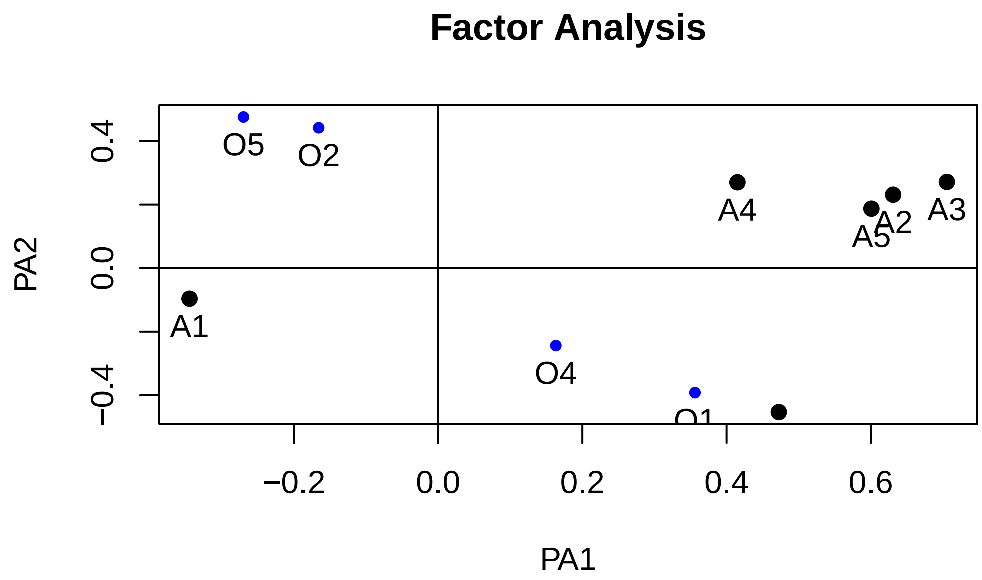
Rotation

Complexity

> **Caution**
>
> The computation of variable complexity and factor complexity here are just for demonstrative purpose. You should not choose a rotation method based on these, because different rotations have different goals and complexity functions.

Unrotated solution

```r
plot(fa_unrotated, labels = c(paste0("O", 1:5), paste0("A", 1:5)))
```




```
# Variable Complexity
lambda_unrotated <- fa_unrotated$loadings
sum(lambda_unrotated[, 1]^2 * lambda_unrotated[, 2]^2)
```

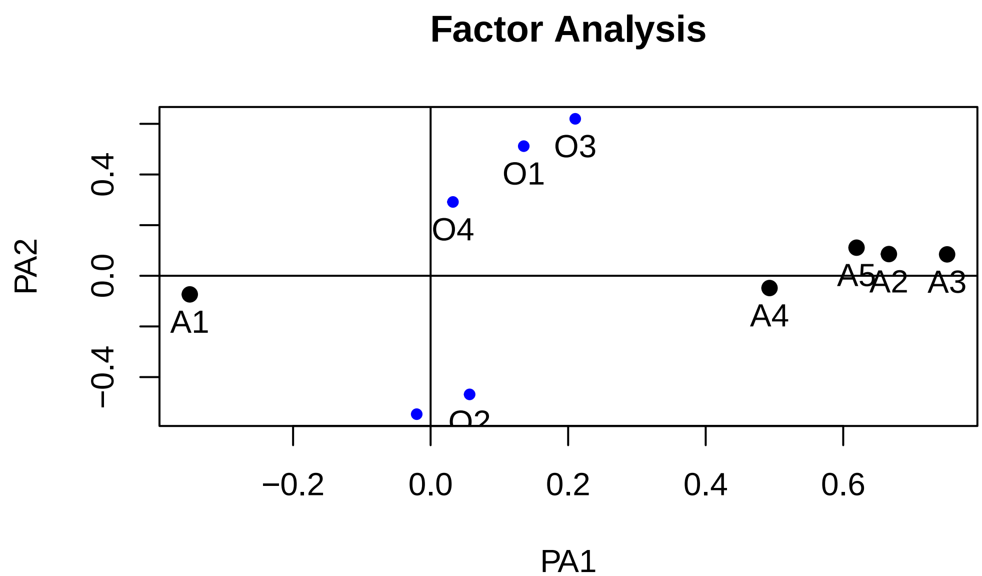
```
[1] 0.172905
```

```
# Factor Complexity
fac_complexity <- 0
for (j in seq_len(ncol(lambda_unrotated))) {
    for (i in seq_len(nrow(lambda_unrotated))) {
        fac_complexity <- fac_complexity + sum(lambda_unrotated[i, j]^2 * lambda_unrotated[-i
    }
}
fac_complexity
```

```
[1] 4.447726
```

Varimax

```r
fa_varimax <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2, fm = "pa", rotate = "varimax")
plot(fa_varimax, labels = c(paste0("O", 1:5), paste0("A", 1:5)))
```




```r
# Variable Complexity
(lambda_varimax <- fa_varimax$loadings)
```

```
Loadings:
   PA1    PA2
O1  0.135  0.512
O2        -0.468
O3  0.210  0.620
O4         0.292
O5        -0.546
A1 -0.350
A2  0.666
A3  0.751
A4  0.493
A5  0.619  0.111
```

```
                 PA1   PA2
SS loadings    1.825 1.284
Proportion Var 0.182 0.128
Cumulative Var 0.182 0.311
```

```
sum(lambda_varimax[, 1]^2 * lambda_varimax[, 2]^2)
```

```
[1] 0.03597018
```

```
# Factor Complexity
fac_complexity <- 0
for (j in seq_len(ncol(lambda_varimax))) {
    for (i in seq_len(nrow(lambda_varimax))) {
        fac_complexity <- fac_complexity + sum(lambda_varimax[i, j]^2 * lambda_varimax[-i, j]
    }
}
fac_complexity
```

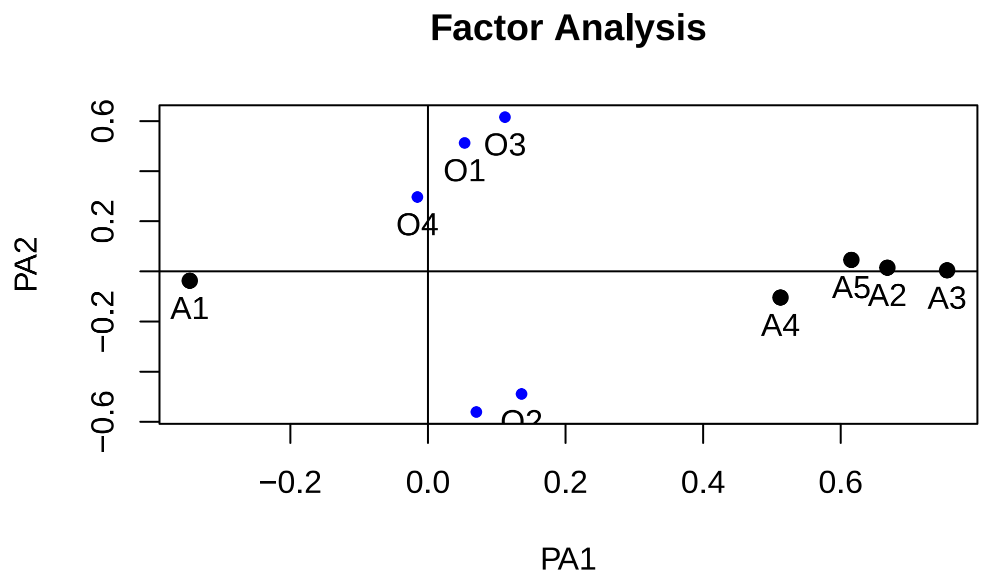
```
[1] 3.877532
```

Oblimin

```
fa_oblimin <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2, fm = "pa", rotate = "oblimin")
```

```
Loading required namespace: GPArotation
```

```
plot(fa_oblimin, labels = c(paste0("O", 1:5), paste0("A", 1:5)))
```




```
# Variable Complexity
(lambda_oblimin <- fa_oblimin$loadings)
```

```
Loadings:
   PA1    PA2   
O1         0.513
O2  0.136 -0.489
O3  0.112  0.616
O4         0.297
O5        -0.561
A1 -0.346       
A2  0.668       
A3  0.755       
A4  0.512 -0.104
A5  0.615       

                 PA1   PA2
SS loadings    1.816 1.300
Proportion Var 0.182 0.130
Cumulative Var 0.182 0.312
```

```
sum(lambda_oblimin[, 1]^2 * lambda_oblimin[, 2]^2)
```

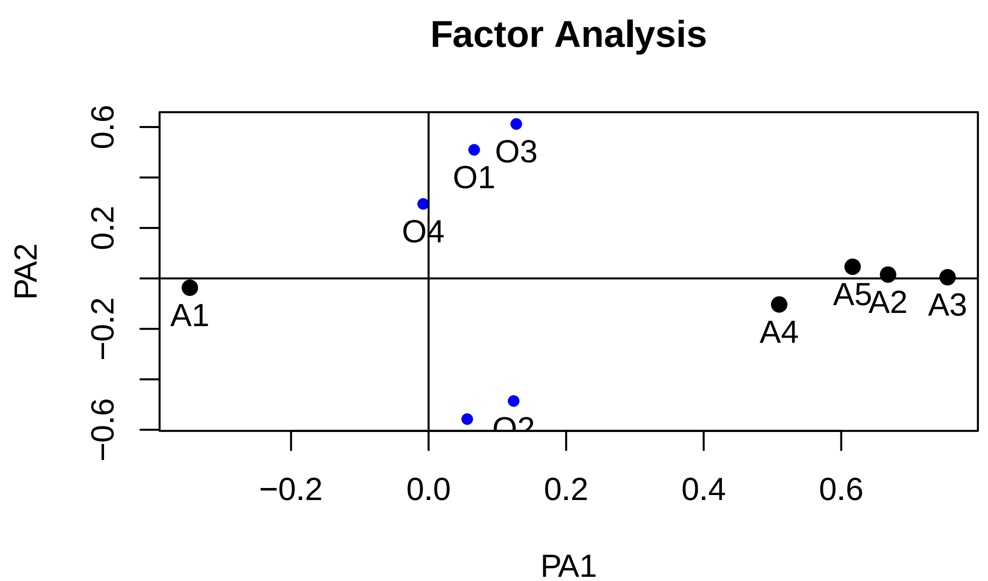
```
[1] 0.01545015
```

```
# Factor Complexity
fac_complexity <- 0
for (j in seq_len(ncol(lambda_oblimin))) {
    for (i in seq_len(nrow(lambda_oblimin))) {
        fac_complexity <- fac_complexity + sum(lambda_oblimin[i, j]^2 * lambda_oblimin[-i, j]
    }
}
fac_complexity
```

```
[1] 3.858235
```

Geomin (Oblique)

```
fa_geomin <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2, fm = "pa", rotate = "geominQ")
plot(fa_geomin, labels = c(paste0("O", 1:5), paste0("A", 1:5)))
```



```r
# Variable Complexity
(lambda_geomin <- fa_geomin$loadings)
```

```

Loadings:
   PA1    PA2   
O1         0.510
O2  0.124 -0.486
O3  0.128  0.612
O4         0.295
O5        -0.558
A1 -0.347       
A2  0.668       
A3  0.755       
A4  0.510 -0.103
A5  0.617       

                 PA1   PA2
SS loadings    1.816 1.283
Proportion Var 0.182 0.128
Cumulative Var 0.182 0.310
```

```r
sum(lambda_geomin[, 1]^2 * lambda_geomin[, 2]^2)
```

```
[1] 0.01569736
```

```r
# Factor Complexity
fac_complexity <- 0
for (j in seq_len(ncol(lambda_geomin))) {
    for (i in seq_len(nrow(lambda_geomin))) {
        fac_complexity <- fac_complexity + sum(lambda_geomin[i, j]^2 * lambda_geomin[-i, j]^
    }
}
fac_complexity
```

```
[1] 3.823991
```

Target Rotation

In practice, we usually have some idea about the factor structure, and we can rotate the loadings to something close to that idea. This is an example of blurring the distinction between EFA and CFA.

Let's say we think that the five O items are for one factor, and the five A items are for another.

```
# We want the first five items to load on factor 1 but not on factor 2,
# and the reverse for the next five items
target_mat <- matrix(
    c(NA, NA, NA, NA, NA, 0, 0, 0, 0, 0,
      0, 0, 0, 0, 0, NA, NA, NA, NA, NA),
    nrow = 10
)
fa_target <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2, fm = "pa",
    rotate = "targetQ", Target = target_mat)
fa_target
```

```
Factor Analysis using method =  pa
Call: psych::fa(r = select(bfi, O1:O5, A1:A5), nfactors = 2, rotate = "targetQ",
    fm = "pa", Target = target_mat)
Standardized loadings (pattern matrix) based upon correlation matrix
     PA2   PA1    h2   u2 com
O1  0.06  0.51 0.280 0.72 1.0
O2  0.13 -0.49 0.223 0.78 1.1
O3  0.12  0.61 0.428 0.57 1.1
O4 -0.01  0.30 0.086 0.91 1.0
O5  0.06 -0.56 0.299 0.70 1.0
A1 -0.35 -0.04 0.128 0.87 1.0
A2  0.67  0.02 0.451 0.55 1.0
A3  0.75  0.01 0.571 0.43 1.0
A4  0.51 -0.10 0.245 0.75 1.1
A5  0.62  0.05 0.396 0.60 1.0

                       PA2  PA1
SS loadings           1.81 1.29
Proportion Var        0.18 0.13
Cumulative Var        0.18 0.31
Proportion Explained  0.58 0.42
```

```
Cumulative Proportion 0.58 1.00

 With factor correlations of 
     PA2  PA1
PA2 1.00 0.25
PA1 0.25 1.00

Mean item complexity =  1
Test of the hypothesis that 2 factors are sufficient.

df null model =  45  with the objective function =  1.58 with Chi Square =  4409.68
df of  the model are 26  and the objective function was  0.12 

The root mean square of the residuals (RMSR) is  0.04 
The df corrected root mean square of the residuals is  0.05 

The harmonic n.obs is  2766 with the empirical chi square  362.29  with prob <  6e-61 
The total n.obs was  2800  with Likelihood Chi Square =  323  with prob <  5.2e-53 

Tucker Lewis Index of factoring reliability =  0.882
RMSEA index =  0.064  and the 90 % confidence intervals are  0.058 0.07
BIC =  116.63
Fit based upon off diagonal values = 0.97
Measures of factor score adequacy             
                                                   PA2  PA1
Correlation of (regression) scores with factors   0.88 0.81
Multiple R square of scores with factors          0.77 0.66
Minimum correlation of possible factor scores     0.54 0.32
```

Percentage of Variance Accounted For

Let's use `oblimin` solution

```r
# Proportion of variance by the first two eigenvalues
ev_oa <- fa_oblimin$e.values
sum(ev_oa[1:2]) / sum(ev_oa)
```

```
[1] 0.4399516
```

The first two eigenvalues account for 44% of the total variance in the data.

Correlation Residual

This does not depend on rotation method.

```
round(fa_unrotated$residual, digits = 2)
```

```
     O1    O2    O3    O4    O5    A1    A2    A3    A4    A5
O1 0.72  0.02  0.05  0.02  0.04  0.10  0.00  0.00  0.02  0.02
O2 0.02  0.78  0.02  0.07  0.07  0.06  0.02  0.00 -0.01  0.02
O3 0.05  0.02  0.57  0.01  0.03  0.06 -0.03  0.01  0.00  0.04
O4 0.02  0.07  0.01  0.91 -0.02 -0.04  0.04 -0.01 -0.04 -0.04
O5 0.04  0.07  0.03 -0.02  0.70  0.07 -0.03  0.01  0.00  0.02
A1 0.10  0.06  0.06 -0.04  0.07  0.87 -0.10  0.00  0.02  0.04
A2 0.00  0.02 -0.03  0.04 -0.03 -0.10  0.55 -0.02  0.01 -0.03
A3 0.00  0.00  0.01 -0.01  0.01  0.00 -0.02  0.43 -0.01  0.03
A4 0.02 -0.01  0.00 -0.04  0.00  0.02  0.01 -0.01  0.75  0.01
A5 0.02  0.02  0.04 -0.04  0.02  0.04 -0.03  0.03  0.01  0.60
```

As a general rule of thumb, researchers should pay attention to residuals with absolute value > .10. In our case, there were a few values close to .10, but it doesn't seem to be strong evidence that there may be an additional factor needed.

Confidence Intervals

The loadings in EFA are quite unstable, especially with rotation. We can obtain confidence intervals using bootstrap, which is available in `psych::fa()` by specifying

```
fa_oblimin_boot <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2,
    n.iter = 200, # number of bootstrap samples
    fm = "pa", rotate = "oblimin")
fa_oblimin_boot
```

```
Factor Analysis with confidence intervals using method = psych::fa(r = select(bfi, O1:O5, A1
    rotate = "oblimin", fm = "pa")
Factor Analysis using method =  pa
Call: psych::fa(r = select(bfi, O1:O5, A1:A5), nfactors = 2, n.iter = 200,
    rotate = "oblimin", fm = "pa")
```

```
Standardized loadings (pattern matrix) based upon correlation matrix
     PA1   PA2    h2   u2 com
O1  0.05  0.51 0.280 0.72 1.0
O2  0.14 -0.49 0.223 0.78 1.2
O3  0.11  0.62 0.428 0.57 1.1
O4 -0.02  0.30 0.086 0.91 1.0
O5  0.07 -0.56 0.299 0.70 1.0
A1 -0.35 -0.04 0.128 0.87 1.0
A2  0.67  0.01 0.451 0.55 1.0
A3  0.75  0.00 0.571 0.43 1.0
A4  0.51 -0.10 0.245 0.75 1.1
A5  0.62  0.05 0.396 0.60 1.0

                       PA1  PA2
SS loadings           1.81 1.30
Proportion Var        0.18 0.13
Cumulative Var        0.18 0.31
Proportion Explained  0.58 0.42
Cumulative Proportion 0.58 1.00

 With factor correlations of
     PA1  PA2
PA1 1.00 0.26
PA2 0.26 1.00

Mean item complexity =  1
Test of the hypothesis that 2 factors are sufficient.

df null model =  45  with the objective function =  1.58 with Chi Square =  4409.68
df of  the model are 26  and the objective function was  0.12

The root mean square of the residuals (RMSR) is  0.04
The df corrected root mean square of the residuals is  0.05

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Tucker Lewis Index of factoring reliability =  0.882
RMSEA index =  0.064  and the 90 % confidence intervals are  0.058 0.07
BIC =  116.63
Fit based upon off diagonal values = 0.97
Measures of factor score adequacy
                                                   PA1  PA2
```

```
Correlation of (regression) scores with factors   0.88 0.81
Multiple R square of scores with factors          0.77 0.66
Minimum correlation of possible factor scores     0.54 0.33

 Coefficients and bootstrapped confidence intervals
     low   PA1 upper   low   PA2 upper
O1  0.02  0.05  0.10  0.46  0.51  0.56
O2  0.10  0.14  0.17 -0.54 -0.49 -0.44
O3  0.08  0.11  0.15  0.56  0.62  0.67
O4 -0.06 -0.02  0.03  0.24  0.30  0.35
O5  0.03  0.07  0.11 -0.61 -0.56 -0.51
A1 -0.39 -0.35 -0.30 -0.10 -0.04  0.01
A2  0.63  0.67  0.71 -0.02  0.01  0.05
A3  0.72  0.75  0.78 -0.02  0.00  0.04
A4  0.47  0.51  0.55 -0.15 -0.10 -0.07
A5  0.57  0.62  0.66  0.00  0.05  0.09

 Interfactor correlations and bootstrapped confidence intervals
        lower estimate upper
PA1-PA2  0.21     0.26  0.32
```

In practice, you should use more bootstrap samples (e.g., 1999). We can do the same for target rotation:

```
fa_target_boot <- psych::fa(
    select(bfi, O1:O5, A1:A5),
    nfactors = 2,
    n.iter = 200, # number of bootstrap samples
    fm = "pa",
    rotate = "targetQ", Target = target_mat)
fa_target_boot
```

```
Factor Analysis with confidence intervals using method = psych::fa(r = select(bfi, O1:O5, A1
    rotate = "targetQ", fm = "pa", Target = target_mat)
Factor Analysis using method =  pa
Call: psych::fa(r = select(bfi, O1:O5, A1:A5), nfactors = 2, n.iter = 200,
    rotate = "targetQ", fm = "pa", Target = target_mat)
Standardized loadings (pattern matrix) based upon correlation matrix
     PA2   PA1    h2   u2 com
O1  0.06  0.51 0.280 0.72 1.0
O2  0.13 -0.49 0.223 0.78 1.1
O3  0.12  0.61 0.428 0.57 1.1
```

```
O4 -0.01  0.30 0.086 0.91 1.0
O5  0.06 -0.56 0.299 0.70 1.0
A1 -0.35 -0.04 0.128 0.87 1.0
A2  0.67  0.02 0.451 0.55 1.0
A3  0.75  0.01 0.571 0.43 1.0
A4  0.51 -0.10 0.245 0.75 1.1
A5  0.62  0.05 0.396 0.60 1.0

                       PA2  PA1
SS loadings           1.81 1.29
Proportion Var        0.18 0.13
Cumulative Var        0.18 0.31
Proportion Explained  0.58 0.42
Cumulative Proportion 0.58 1.00

 With factor correlations of
     PA2  PA1
PA2 1.00 0.25
PA1 0.25 1.00

Mean item complexity =  1
Test of the hypothesis that 2 factors are sufficient.

df null model =  45  with the objective function =  1.58 with Chi Square =  4409.68
df of  the model are 26  and the objective function was  0.12

The root mean square of the residuals (RMSR) is  0.04
The df corrected root mean square of the residuals is  0.05

The harmonic n.obs is  2766 with the empirical chi square  362.29  with prob <  6e-61
The total n.obs was  2800  with Likelihood Chi Square =  323  with prob <  5.2e-53

Tucker Lewis Index of factoring reliability =  0.882
RMSEA index =  0.064  and the 90 % confidence intervals are  0.058 0.07
BIC =  116.63
Fit based upon off diagonal values = 0.97
Measures of factor score adequacy
                                                   PA2  PA1
Correlation of (regression) scores with factors   0.88 0.81
Multiple R square of scores with factors          0.77 0.66
Minimum correlation of possible factor scores     0.54 0.32

 Coefficients and bootstrapped confidence intervals
```

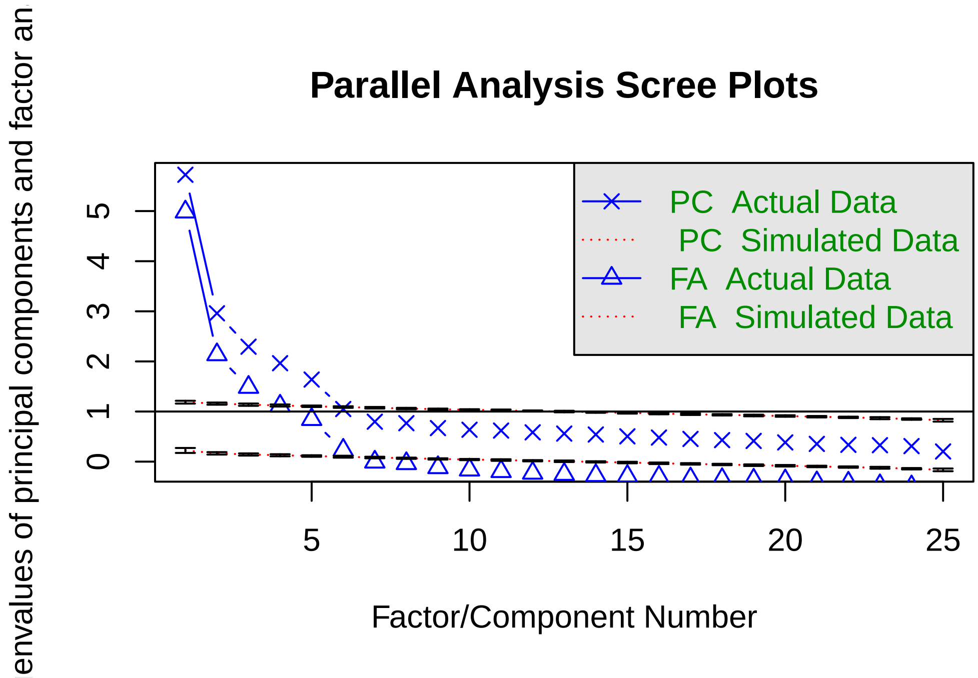
```
     low   PA2 upper   low   PA1 upper
O1  0.02  0.06  0.10  0.47  0.51  0.56
O2  0.09  0.13  0.16 -0.53 -0.49 -0.43
O3  0.08  0.12  0.16  0.57  0.61  0.66
O4 -0.06 -0.01  0.03  0.24  0.30  0.35
O5  0.02  0.06  0.10 -0.61 -0.56 -0.51
A1 -0.40 -0.35 -0.30 -0.08 -0.04  0.01
A2  0.63  0.67  0.71 -0.02  0.02  0.05
A3  0.72  0.75  0.79 -0.02  0.01  0.04
A4  0.47  0.51  0.55 -0.14 -0.10 -0.07
A5  0.58  0.62  0.66  0.01  0.05  0.09

 Interfactor correlations and bootstrapped confidence intervals
        lower estimate upper
PA2-PA1   0.2     0.25   0.3
```

Full BFI

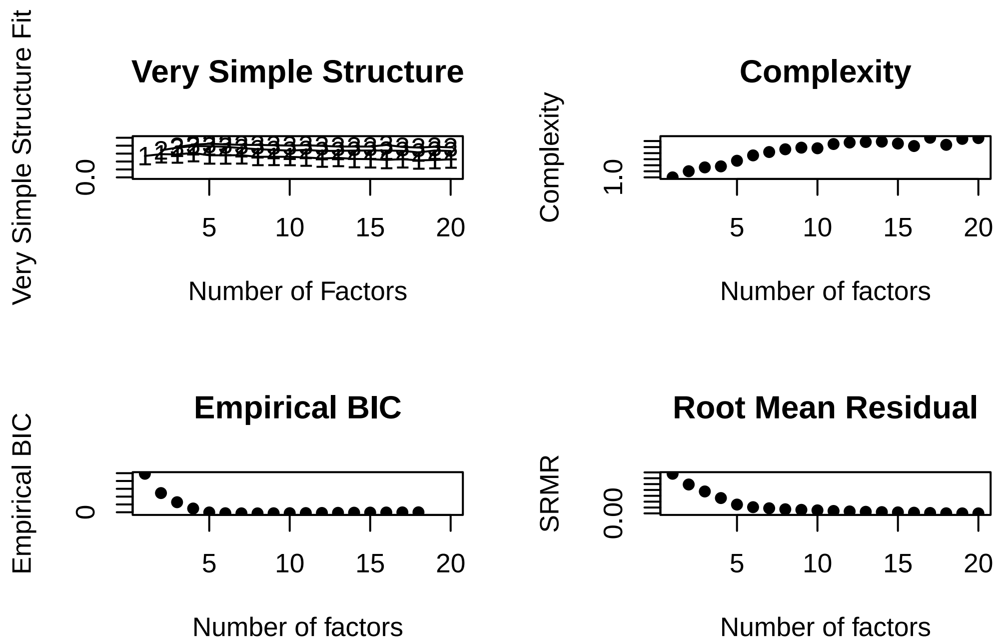
Target rotation with BFI, with polychoric correlation

```
pcorr_bfi <- lavaan::lavCor(select(bfi, A1:O5), ordered = TRUE)
n_pcorr_bfi <-
    select(bfi, A1:O5) |>
    drop_na() |>
    nrow()
# Parallel analysis
psych::fa.parallel(
    pcorr_bfi,
    n.obs = n_pcorr_bfi,
    fm = "pa",
    error.bars = TRUE
)
```




```
Parallel analysis suggests that the number of factors =  6  and the number of components =  5
```

```
# Other methods
nfactors(pcorr_bfi, n.obs = n_pcorr_bfi)
```




```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of Although the vss.max shows  4  factors, it is probal
VSS complexity 2 achieves a maximimum of 0.79  with  4  factors
The Velicer MAP achieves a minimum of 0.02  with  5  factors
Empirical BIC achieves a minimum of  -711.98  with  8  factors
Sample Size adjusted BIC achieves a minimum of  -125.98  with  12  factors

Statistics by number of factors
   vss1 vss2   map dof   chisq     prob sqresid  fit  RMSEA   BIC   SABIC
1  0.54 0.00 0.032 275 1.4e+04  0.0e+00    27.3 0.54 0.1438 11980 12853.7
2  0.59 0.68 0.026 251 9.3e+03  0.0e+00    18.8 0.68 0.1213  7295  8092.2
3  0.58 0.75 0.023 228 6.6e+03  0.0e+00    13.9 0.77 0.1074  4860  5584.3
4  0.62 0.79 0.019 206 4.4e+03  0.0e+00    10.1 0.83 0.0912  2774  3428.8
5  0.56 0.79 0.016 185 2.2e+03  0.0e+00     7.6 0.87 0.0673   783  1370.9
6  0.56 0.77 0.017 165 1.4e+03 1.7e-188     6.6 0.89 0.0547    82   606.2
7  0.56 0.74 0.021 146 9.6e+02 1.8e-120     6.2 0.90 0.0479  -175   288.7
8  0.52 0.71 0.023 128 7.1e+02  1.2e-80     5.7 0.90 0.0430  -293   114.0
9  0.52 0.69 0.028 111 5.2e+02  5.4e-54     5.5 0.91 0.0388  -347     6.1
10 0.52 0.68 0.033  95 3.9e+02  1.0e-37     5.0 0.92 0.0358  -349   -47.0
11 0.50 0.70 0.040  80 2.5e+02  4.7e-20     5.0 0.92 0.0299  -369  -115.2
```

```
12 0.48 0.67 0.048  66 1.8e+02  2.3e-12     4.7 0.92 0.0265  -336  -126.0
13 0.49 0.67 0.058  53 1.3e+02  3.2e-08     4.5 0.92 0.0242  -285  -116.3
14 0.47 0.68 0.066  41 9.0e+01  1.8e-05     4.2 0.93 0.0220  -230   -99.9
15 0.46 0.68 0.081  30 6.0e+01  9.3e-04     3.7 0.94 0.0202  -174   -78.7
16 0.44 0.68 0.097  20 2.4e+01  2.5e-01     3.3 0.94 0.0089  -132   -68.5
17 0.46 0.67 0.114  11 7.8e+00  7.3e-01     3.7 0.94 0.0000   -78   -43.0
18 0.43 0.63 0.134   3 4.5e-01  9.3e-01     3.5 0.94 0.0000   -23   -13.4
19 0.44 0.68 0.157  -4 3.8e-04        NA     3.7 0.94     NA    NA      NA
20 0.45 0.67 0.199 -10 8.4e-05        NA     3.7 0.94     NA    NA      NA
   complex  eChisq    SRMR  eCRMS  eBIC
1      1.0 2.7e+04 1.4e-01 0.1414 24627
2      1.2 1.4e+04 9.9e-02 0.1078 12266
3      1.3 8.2e+03 7.5e-02 0.0859  6414
4      1.4 4.0e+03 5.2e-02 0.0631  2392
5      1.5 1.3e+03 3.0e-02 0.0383  -121
6      1.7 6.6e+02 2.1e-02 0.0286  -631
7      1.8 4.4e+02 1.7e-02 0.0249  -697
8      1.9 2.9e+02 1.4e-02 0.0214  -712
9      2.0 2.1e+02 1.2e-02 0.0198  -653
10     2.0 1.6e+02 1.0e-02 0.0184  -585
11     2.1 9.7e+01 8.2e-03 0.0158  -526
12     2.2 6.4e+01 6.6e-03 0.0141  -451
13     2.2 4.3e+01 5.4e-03 0.0129  -370
14     2.2 2.7e+01 4.3e-03 0.0117  -292
15     2.1 1.7e+01 3.5e-03 0.0109  -216
16     2.0 7.6e+00 2.3e-03 0.0088  -148
17     2.3 2.6e+00 1.3e-03 0.0070   -83
18     2.1 1.0e-01 2.6e-04 0.0026   -23
19     2.3 7.5e-05 7.2e-06     NA    NA
20     2.3 1.8e-05 3.5e-06     NA    NA
```

```r
# Target rotation
target_mat_bfi <- matrix(0, nrow = 25, ncol = 5)
target_mat_bfi[1:5, 1] <- NA
target_mat_bfi[6:10, 2] <- NA
target_mat_bfi[11:15, 3] <- NA
target_mat_bfi[16:20, 4] <- NA
target_mat_bfi[21:25, 5] <- NA
fa_target_bfi <- psych::fa(
    pcorr_bfi, n.obs = n_pcorr_bfi,
    nfactors = 5, fm = "pa",
    rotate = "targetQ", Target = target_mat_bfi)
```

```r
fa_target_bfi
```

```
Factor Analysis using method =  pa
Call: psych::fa(r = pcorr_bfi, nfactors = 5, n.obs = n_pcorr_bfi, rotate = "targetQ",
    fm = "pa", Target = target_mat_bfi)
Standardized loadings (pattern matrix) based upon correlation matrix
     PA4   PA3   PA2   PA1   PA5   h2   u2 com
A1  0.15  0.18  0.07 -0.51 -0.09 0.26 0.74 1.5
A2  0.03  0.05  0.07  0.69  0.03 0.54 0.46 1.0
A3  0.02  0.18  0.02  0.69  0.04 0.61 0.39 1.1
A4 -0.02  0.10  0.21  0.47 -0.18 0.35 0.65 1.9
A5 -0.11  0.28  0.00  0.55  0.04 0.53 0.47 1.6
C1  0.07 -0.02  0.59  0.00  0.17 0.40 0.60 1.2
C2  0.18 -0.05  0.71  0.06  0.06 0.51 0.49 1.2
C3  0.04 -0.08  0.61  0.10 -0.07 0.36 0.64 1.1
C4  0.17  0.03 -0.69  0.01 -0.03 0.55 0.45 1.1
C5  0.21 -0.10 -0.60  0.01  0.12 0.49 0.51 1.4
E1 -0.01 -0.62  0.13 -0.07 -0.02 0.39 0.61 1.1
E2  0.17 -0.71 -0.01 -0.04  0.01 0.61 0.39 1.1
E3  0.07  0.51 -0.02  0.22  0.27 0.50 0.50 2.0
E4 -0.05  0.64  0.00  0.27 -0.13 0.60 0.40 1.4
E5  0.13  0.50  0.28  0.01  0.17 0.46 0.54 2.0
N1  0.86  0.21  0.01 -0.19 -0.08 0.74 0.26 1.2
N2  0.81  0.14  0.02 -0.16  0.00 0.66 0.34 1.1
N3  0.77  0.00 -0.03  0.02  0.01 0.59 0.41 1.0
N4  0.56 -0.33 -0.13  0.08  0.13 0.55 0.45 1.9
N5  0.56 -0.17  0.00  0.20 -0.15 0.40 0.60 1.6
O1  0.00  0.18  0.06  0.00  0.55 0.39 0.61 1.3
O2  0.20  0.02 -0.08  0.15 -0.50 0.30 0.70 1.6
O3  0.02  0.27 -0.01  0.05  0.63 0.54 0.46 1.4
O4  0.19 -0.28 -0.04  0.19  0.48 0.35 0.65 2.4
O5  0.11  0.05 -0.04  0.04 -0.59 0.36 0.64 1.1

                       PA4  PA3  PA2  PA1  PA5
SS loadings           2.96 2.61 2.33 2.26 1.85
Proportion Var        0.12 0.10 0.09 0.09 0.07
Cumulative Var        0.12 0.22 0.32 0.41 0.48
Proportion Explained  0.25 0.22 0.19 0.19 0.15
Cumulative Proportion 0.25 0.46 0.66 0.85 1.00

 With factor correlations of
      PA4   PA3   PA2   PA1  PA5
```

```
PA4  1.00 -0.21 -0.21 -0.09 0.02
PA3 -0.21  1.00  0.29  0.34 0.16
PA2 -0.21  0.29  1.00  0.23 0.20
PA1 -0.09  0.34  0.23  1.00 0.16
PA5  0.02  0.16  0.20  0.16 1.00

Mean item complexity =  1.4
Test of the hypothesis that 5 factors are sufficient.

df null model =  300  with the objective function =  9.59 with Chi Square =  23261.83
df of  the model are 185  and the objective function was  0.92

The root mean square of the residuals (RMSR) is  0.03
The df corrected root mean square of the residuals is  0.04

The harmonic n.obs is  2436 with the empirical chi square  1322.13  with prob <  8.1e-171
The total n.obs was  2436  with Likelihood Chi Square =  2226.01  with prob <  0

Tucker Lewis Index of factoring reliability =  0.856
RMSEA index =  0.067  and the 90 % confidence intervals are  0.065 0.07
BIC =  783.36
Fit based upon off diagonal values = 0.98
Measures of factor score adequacy
                                                   PA4  PA3  PA2  PA1  PA5
Correlation of (regression) scores with factors   0.94 0.92 0.90 0.90 0.87
Multiple R square of scores with factors          0.89 0.84 0.81 0.81 0.76
Minimum correlation of possible factor scores     0.77 0.68 0.63 0.62 0.52
```

Using the `lavaan` R package

More information at https://lavaan.ugent.be/tutorial/efa.html

```r
# One- to Six-factor models
efa_fit <- efa(
    select(bfi, A1:O5),
    nfactors = 1:6,
    ordered = TRUE  # only needed for ordinal data
)
summary(efa_fit)
```

```
This is lavaan 0.6-19 -- running exploratory factor analysis
```

```
  Estimator                                                 DWLS
  Rotation method                                 GEOMIN OBLIQUE
  Geomin epsilon                                           0.001
  Rotation algorithm (rstarts)                          GPA (30)
  Standardized metric                                       TRUE
  Row weights                                               None

                                                            Used       Total
  Number of observations                                    2436        2800

Overview models:
                   chisq  df pvalue   cfi rmsea
  nfactors = 1 16233.810 275      0 0.363 0.148
  nfactors = 2 10653.913 251      0 0.609 0.122
  nfactors = 3  7391.517 228      0 0.723 0.107
  nfactors = 4  4711.331 206      0 0.819 0.091
  nfactors = 5  1856.317 185      0 0.914 0.066
  nfactors = 6  1058.461 165      0 0.950 0.054

Eigenvalues correlation matrix:

    ev1     ev2     ev3     ev4     ev5     ev6     ev7     ev8     ev9    ev10
  5.725   2.960   2.294   1.964   1.638   1.050   0.797   0.767   0.669   0.637
   ev11    ev12    ev13    ev14    ev15    ev16    ev17    ev18    ev19    ev20
  0.620   0.584   0.560   0.541   0.504   0.480   0.453   0.429   0.413   0.383
   ev21    ev22    ev23    ev24    ev25
  0.354   0.335   0.328   0.310   0.203

Number of factors:  1

Standardized loadings: (* = significant at 1% level)

       f1       unique.var   communalities
A1      .*           0.930           0.070
A2 -0.503*           0.747           0.253
A3 -0.587*           0.655           0.345
A4 -0.432*           0.813           0.187
A5 -0.623*           0.611           0.389
C1 -0.377*           0.858           0.142
C2 -0.360*           0.871           0.129
C3 -0.330*           0.891           0.109
C4  0.510*           0.740           0.260
```

C5	0.510*	0.740	0.260
E1	0.418*	0.825	0.175
E2	0.626*	0.609	0.391
E3	-0.542*	0.706	0.294
E4	-0.624*	0.610	0.390
E5	-0.497*	0.753	0.247
N1	0.701*	0.508	0.492
N2	0.674*	0.546	0.454
N3	0.547*	0.700	0.300
N4	0.559*	0.688	0.312
N5	0.407*	0.834	0.166
O1	-0.355*	0.874	0.126
O2	.*	0.947	0.053
O3	-0.412*	0.830	0.170
O4	*	0.997	0.003
O5	.*	0.947	0.053

	f1
Sum of squared loadings	5.769
Proportion of total	1.000
Proportion var	0.231
Cumulative var	0.231

Number of factors: 2

Standardized loadings: (* = significant at 1% level)

	f1	f2	unique.var	communalities
A1	.*	*	0.925	0.075
A2	0.591*		0.656	0.344
A3	0.672*		0.554	0.446
A4	0.441*	*	0.784	0.216
A5	0.641*	.*	0.558	0.442
C1	0.431*		0.811	0.189
C2	0.451*		0.800	0.200
C3	0.338*	*	0.869	0.131
C4	-0.420*	.*	0.707	0.293
C5	-0.376*	0.343*	0.700	0.300
E1	-0.467*		0.776	0.224
E2	-0.579*	.*	0.570	0.430
E3	0.663*	*	0.571	0.429
E4	0.624*	.*	0.563	0.437
E5	0.615*	*	0.629	0.371

```
N1          0.859*           0.264           0.736
N2          0.845*           0.292           0.708
N3          0.750*           0.437           0.563
N4      .*  0.614*           0.539           0.461
N5       *  0.525*           0.705           0.295
O1  0.456*       *           0.797           0.203
O2      .*       *           0.944           0.056
O3  0.542*      .*           0.713           0.287
O4      .*  0.304*           0.902           0.098
O5      .*                   0.925           0.075

                              f1    f2 total
Sum of sq (obliq) loadings 4.821 3.188 8.009
Proportion of total        0.602 0.398 1.000
Proportion var             0.193 0.128 0.320
Cumulative var             0.193 0.320 0.320

Factor correlations: (* = significant at 1% level)

       f1      f2
f1  1.000
f2 -0.161*  1.000

Number of factors:  3

Standardized loadings: (* = significant at 1% level)

       f1      f2      f3       unique.var   communalities
A1              .*       *           0.902           0.098
A2          0.607*                   0.615           0.385
A3          0.715*                   0.483           0.517
A4       *  0.433*      .*           0.767           0.233
A5          0.681*      .*           0.496           0.504
C1  0.621*               *           0.604           0.396
C2  0.648*                           0.575           0.425
C3  0.474*              .*           0.751           0.249
C4 -0.614*          0.371*           0.493           0.507
C5 -0.444*       *  0.406*           0.608           0.392
E1       * -0.572*                   0.685           0.315
E2         -0.619*      .*           0.510           0.490
E3      .*  0.615*                   0.548           0.452
E4       *  0.720*      .*           0.451           0.549
E5  0.340*  0.442*                   0.611           0.389
```

N1			0.859*	0.265	0.735
N2	*		0.840*	0.290	0.710
N3			0.751*	0.438	0.562
N4		.*	0.625*	0.533	0.467
N5	*		0.538*	0.700	0.300
O1	0.400*	.*		0.746	0.254
O2	-0.373*	*	.*	0.849	0.151
O3	0.386*	0.331*	*	0.678	0.322
O4	.*		.*	0.864	0.136
O5	-0.393*			0.841	0.159

	f2	f3	f1	total
Sum of sq (obliq) loadings	3.814	3.322	2.561	9.697
Proportion of total	0.393	0.343	0.264	1.000
Proportion var	0.153	0.133	0.102	0.388
Cumulative var	0.153	0.285	0.388	0.388

Factor correlations: (* = significant at 1% level)

	f1	f2	f3
f1	1.000		
f2	0.256*	1.000	
f3	-0.003	-0.143*	1.000

Number of factors: 4

Standardized loadings: (* = significant at 1% level)

	f1	f2	f3	f4	unique.var	communalities
A1		.*	*		0.904	0.096
A2	.*	0.592*			0.592	0.408
A3	.*	0.702*			0.470	0.530
A4	.*	0.423*		.*	0.670	0.330
A5		0.672*	.*		0.495	0.505
C1	0.563*			.*	0.593	0.407
C2	0.679*		*	.*	0.495	0.505
C3	0.592*				0.644	0.356
C4	-0.657*		.*	.*	0.447	0.553
C5	-0.569*	*	0.307*		0.524	0.476
E1	.*	-0.571*		*	0.675	0.325
E2		-0.616*	.*	.*	0.506	0.494
E3		0.626*		0.326*	0.506	0.494
E4		0.715*	.*		0.442	0.558

```
E5      .*  0.441*              .*           0.610           0.390
N1                0.866*         *           0.256           0.744
N2                0.843*                     0.291           0.709
N3                0.751*                     0.435           0.565
N4      .*      .*  0.606*                   0.526           0.474
N5                0.567*        .*           0.651           0.349
O1              .*          0.571*           0.607           0.393
O2                      .* -0.523*           0.689           0.311
O3          0.358*          0.648*           0.457           0.543
O4                      .*  0.373*           0.792           0.208
O5                      .* -0.580*           0.657           0.343

                              f2    f3    f1    f4  total
Sum of sq (obliq) loadings 3.747 3.187 2.240 1.893 11.066
Proportion of total        0.339 0.288 0.202 0.171  1.000
Proportion var             0.150 0.127 0.090 0.076  0.443
Cumulative var             0.150 0.277 0.367 0.443  0.443

Factor correlations: (* = significant at 1% level)

      f1      f2      f3      f4
f1   1.000
f2  0.237*   1.000
f3 -0.090* -0.137*   1.000
f4  0.096*   0.006   0.057   1.000

Number of factors:  5

Standardized loadings: (* = significant at 1% level)

      f1      f2      f3      f4      f5      unique.var   communalities
A1 -0.472*              .*                         0.744           0.256
A2  0.466*       *  0.565*      .*                 0.467           0.533
A3  0.440*          0.685*       *                 0.383           0.617
A4      .*      .*  0.401*              .*         0.643           0.357
A5  0.346*          0.677*                         0.459           0.541
C1          0.578*                      .*         0.593           0.407
C2          0.687*              .*      .*         0.492           0.508
C3          0.592*                                 0.643           0.357
C4         -0.673*              .*      .*         0.449           0.551
C5         -0.585*       *  0.318*                 0.505           0.495
E1      .*      .* -0.604*       *                 0.598           0.402
E2      .*         -0.653*      .*                 0.391           0.609
```

```
E3                  0.665*              .*           0.500           0.500
E4                  0.743*      .*      .*           0.391           0.609
E5      .*      .*  0.477*              .*           0.542           0.458
N1 -0.530*                  0.787*       *           0.193           0.807
N2 -0.471*                  0.760*                   0.284           0.716
N3      .*                  0.756*                   0.418           0.582
N4              .*      .*  0.655*       *           0.441           0.559
N5                          0.603*      .*           0.601           0.399
O1                      .*          0.545*           0.609           0.391
O2               *              .* -0.518*           0.689           0.311
O3                  0.409*          0.610*           0.461           0.539
O4      .*                  0.301*  0.435*           0.656           0.344
O5                               * -0.584*           0.643           0.357

                              f3    f4    f2    f5    f1  total
Sum of sq (obliq) loadings 3.832 3.012 2.224 1.795 1.344 12.207
Proportion of total        0.314 0.247 0.182 0.147 0.110  1.000
Proportion var             0.153 0.120 0.089 0.072 0.054  0.488
Cumulative var             0.153 0.274 0.363 0.435 0.488  0.488

Factor correlations: (* = significant at 1% level)

       f1      f2      f3      f4      f5
f1  1.000
f2  0.078   1.000
f3 -0.100   0.293*  1.000
f4  0.113  -0.050  -0.130*  1.000
f5  0.018   0.011   0.008   0.025   1.000

Number of factors:  6

Standardized loadings: (* = significant at 1% level)

       f1      f2      f3      f4      f5      f6       unique.var
A1 -0.607*      .*                      .*                   0.605
A2  0.648*          0.336*      .*                           0.401
A3  0.504*          0.539*               *                   0.392
A4  0.314*      .*      .*                      .*           0.646
A5      .*          0.627*      .*                           0.432
C1          0.598*                              .*           0.583
C2          0.733*                      .*                   0.444
C3       *  0.586*                              .*           0.648
C4      .* -0.649*      .*      .*  0.392*                   0.348
```

```
C5          -0.552*                   0.337*      .*                 0.515
E1               .* -0.548* -0.382*                                  0.553
E2                  -0.621*      .*      .*                          0.380
E3                   0.728*              .*      .*                  0.457
E4                   0.759*      .*              .*                  0.362
E5               .*  0.478*      .*              .*                  0.544
N1                           0.595*  0.756*       *                  0.198
N2                           0.578*  0.700*                          0.258
N3                               .*  0.749*                          0.412
N4       .*       *      .*          0.630*      .*                  0.436
N5       .*                       *  0.610*      .*                  0.595
O1                *  0.364*                  0.534*                  0.585
O2                                   0.313* -0.514*                  0.661
O3                   0.467*                  0.607*                  0.459
O4       .*                      .*      .*  0.456*                  0.655
O5       .*                              .* -0.610*                  0.575
     communalities
A1           0.395
A2           0.599
A3           0.608
A4           0.354
A5           0.568
C1           0.417
C2           0.556
C3           0.352
C4           0.652
C5           0.485
E1           0.447
E2           0.620
E3           0.543
E4           0.638
E5           0.456
N1           0.802
N2           0.742
N3           0.588
N4           0.564
N5           0.405
O1           0.415
O2           0.339
O3           0.541
O4           0.345
O5           0.425
```

```
                              f3    f5    f2    f6    f1    f4  total
Sum of sq (obliq) loadings 3.451 2.971 2.228 1.685 1.485 1.034 12.856
Proportion of total        0.268 0.231 0.173 0.131 0.116 0.080  1.000
Proportion var             0.138 0.119 0.089 0.067 0.059 0.041  0.514
Cumulative var             0.138 0.257 0.346 0.413 0.473 0.514  0.514

Factor correlations: (* = significant at 1% level)

       f1      f2      f3      f4      f5      f6
f1  1.000
f2  0.177*  1.000
f3  0.145*  0.268*  1.000
f4 -0.092   0.034  -0.032   1.000
f5 -0.015  -0.110  -0.289* -0.121   1.000
f6  0.022   0.164* -0.089   0.121*  0.120*  1.000
```

With target rotation

```
# Target rotation (using the same matrix defined above)
efa5fac_fit <- efa(
    select(bfi, A1:O5),
    nfactors = 5,
    ordered = TRUE,
    rotation = "target",
    rotation.args = list(
        target = target_mat_bfi
    )
)
summary(efa5fac_fit)
```

```
This is lavaan 0.6-19 -- running exploratory factor analysis

  Estimator                                       DWLS
  Rotation method                          PST OBLIQUE
  Rotation algorithm (rstarts)                GPA (30)
  Standardized metric                             TRUE
  Row weights                                     None

                                                  Used       Total
  Number of observations                          2436        2800

Fit measures:
```

```
                 chisq  df pvalue   cfi rmsea
  nfactors = 5 1856.317 185      0 0.914 0.066

Eigenvalues correlation matrix:

    ev1     ev2     ev3     ev4     ev5     ev6     ev7     ev8     ev9    ev10
  5.725   2.960   2.294   1.964   1.638   1.050   0.797   0.767   0.669   0.637
   ev11    ev12    ev13    ev14    ev15    ev16    ev17    ev18    ev19    ev20
  0.620   0.584   0.560   0.541   0.504   0.480   0.453   0.429   0.413   0.383
   ev21    ev22    ev23    ev24    ev25
  0.354   0.335   0.328   0.310   0.203

Standardized loadings: (* = significant at 1% level)

       f1      f2      f3      f4      f5       unique.var   communalities
A1 -0.502*       *      .*      .*       *           0.744           0.256
A2  0.684*       *       *                           0.467           0.533
A3  0.693*              .*               *           0.383           0.617
A4  0.467*      .*      .*              .*           0.643           0.357
A5  0.552*              .*      .*                   0.459           0.541
C1          0.595*               *      .*           0.593           0.407
C2       *  0.705*       *      .*       *           0.492           0.508
C3      .*  0.606*       *       *       *           0.643           0.357
C4         -0.692*              .*                   0.449           0.551
C5         -0.604*      .*      .*      .*           0.505           0.495
E1       *      .* -0.636*                           0.598           0.402
E2       *         -0.710*      .*                   0.391           0.609
E3      .*          0.497*       *      .*           0.500           0.500
E4      .*          0.648*       *      .*           0.391           0.609
E5              .*  0.488*      .*      .*           0.542           0.458
N1      .*              .*  0.896*       *           0.193           0.807
N2      .*              .*  0.848*                   0.284           0.716
N3       *       *          0.749*                   0.418           0.582
N4       *      .* -0.341*  0.555*      .*           0.441           0.559
N5      .*              .*  0.548*      .*           0.601           0.399
O1               *      .*          0.550*           0.609           0.391
O2      .*       *              .* -0.508*           0.689           0.311
O3       *              .*          0.630*           0.461           0.539
O4      .*              .*      .*  0.474*           0.656           0.344
O5       *                      .* -0.591*           0.643           0.357

                              f4    f3    f2    f1    f5  total
Sum of sq (obliq) loadings 3.029 2.650 2.368 2.293 1.868 12.207
```

```
Proportion of total        0.248 0.217 0.194 0.188 0.153  1.000
Proportion var             0.121 0.106 0.095 0.092 0.075  0.488
Cumulative var             0.121 0.227 0.322 0.414 0.488  0.488

Factor correlations: (* = significant at 1% level)

       f1      f2      f3      f4      f5
f1  1.000
f2  0.216*  1.000
f3  0.318*  0.284*  1.000
f4 -0.087* -0.206* -0.213*  1.000
f5  0.163*  0.204*  0.148*  0.020   1.000
```

APA Table

Openness and Agreeableness

```
fa_oblimin$loadings |>
    unclass() |>
    as.data.frame() |>
    tibble::rownames_to_column() |>
    flextable::flextable() |>
    bold(i = ~ abs(PA1) >= .30, j = ~ PA1) |>
    bold(i = ~ abs(PA2) >= .30, j = ~ PA2) |>
    set_formatter(
        PA1 = rmlead0,
        PA2 = rmlead0
    ) |>
    set_header_labels(values = c("item", "1", "2")) |>
    add_header_row(values = c("", "Factor loading"), colwidths = c(1, 2)) |>
    align(i = 1, align = "center", part = "header")
```

	Factor loading	
item	1	2
O1	.05	**.51**
O2	.14	**-.49**
O3	.11	**.62**
O4	-.02	.30

	Factor loading	
item	1	2
O5	.07	**-.56**
A1	**-.35**	-.04
A2	**.67**	.01
A3	**.75**	.00
A4	**.51**	-.10
A5	**.62**	.05

25-item from IPIP

```
fa_target_bfi$loadings |>
    unclass() |>
    as.data.frame() |>
    tibble::rownames_to_column() |>
    flextable::flextable() |>
    bold(i = ~ abs(PA1) >= .30, j = ~ PA1) |>
    bold(i = ~ abs(PA2) >= .30, j = ~ PA2) |>
    bold(i = ~ abs(PA3) >= .30, j = ~ PA3) |>
    bold(i = ~ abs(PA4) >= .30, j = ~ PA4) |>
    bold(i = ~ abs(PA5) >= .30, j = ~ PA5) |>
    set_formatter(
        PA1 = rmlead0,
        PA2 = rmlead0,
        PA3 = rmlead0,
        PA4 = rmlead0,
        PA5 = rmlead0
    ) |>
    set_header_labels(values = c("item", "1", "2", "3", "4", "5")) |>
    add_header_row(values = c("", "Factor loading"), colwidths = c(1, 5)) |>
    align(i = 1, align = "center", part = "header")
```

	Factor loading				
item	1	2	3	4	5
A1	.15	.18	.07	**-.51**	-.09

	Factor loading				
item	1	2	3	4	5
A2	.03	.05	.07	**.69**	.03
A3	.02	.18	.02	**.69**	.04
A4	-.02	.10	.21	**.47**	-.18
A5	-.11	.28	-.00	**.55**	.04
C1	.07	-.02	**.59**	-.00	.17
C2	.18	-.05	**.71**	.06	.06
C3	.04	-.08	**.61**	.10	-.07
C4	.17	.03	**-.69**	.01	-.03
C5	.21	-.10	**-.60**	.01	.12
E1	-.01	**-.62**	.13	-.07	-.02
E2	.17	**-.71**	-.01	-.04	.01
E3	.07	**.51**	-.02	.22	.27
E4	-.05	**.64**	-.00	.27	-.13
E5	.13	**.50**	.28	.01	.17
N1	**.86**	.21	.01	-.19	-.08
N2	**.81**	.14	.02	-.16	.00
N3	**.77**	.00	-.03	.02	.01
N4	**.56**	**-.33**	-.13	.08	.13
N5	**.56**	-.17	-.00	.20	-.15
O1	-.00	.18	.06	-.00	**.55**
O2	.20	.02	-.08	.15	**-.50**
O3	.02	.27	-.01	.05	**.63**
O4	.19	-.28	-.04	.19	**.48**
O5	.11	.05	-.04	.04	**-.59**