Item Response Theory

```
library(mirt)
```

Rasch Model

```
data(LSAT7, package = "mirt")
dat <- expand.table(LSAT7)
```

```
m_rasch <- mirt(dat, model = 1, itemtype = "Rasch")
```

```
Iteration: 1, Log-Lik: -2664.942, Max-Change: 0.01075
Iteration: 2, Log-Lik: -2664.913, Max-Change: 0.00257
Iteration: 3, Log-Lik: -2664.910, Max-Change: 0.00210
Iteration: 4, Log-Lik: -2664.908, Max-Change: 0.00179
Iteration: 5, Log-Lik: -2664.906, Max-Change: 0.00156
Iteration: 6, Log-Lik: -2664.905, Max-Change: 0.00137
Iteration: 7, Log-Lik: -2664.904, Max-Change: 0.00141
Iteration: 8, Log-Lik: -2664.903, Max-Change: 0.00110
Iteration: 9, Log-Lik: -2664.903, Max-Change: 0.00095
Iteration: 10, Log-Lik: -2664.902, Max-Change: 0.00086
Iteration: 11, Log-Lik: -2664.902, Max-Change: 0.00072
Iteration: 12, Log-Lik: -2664.902, Max-Change: 0.00063
Iteration: 13, Log-Lik: -2664.902, Max-Change: 0.00065
Iteration: 14, Log-Lik: -2664.901, Max-Change: 0.00051
Iteration: 15, Log-Lik: -2664.901, Max-Change: 0.00044
Iteration: 16, Log-Lik: -2664.901, Max-Change: 0.00040
Iteration: 17, Log-Lik: -2664.901, Max-Change: 0.00034
Iteration: 18, Log-Lik: -2664.901, Max-Change: 0.00029
Iteration: 19, Log-Lik: -2664.901, Max-Change: 0.00030
```

```
Iteration: 20, Log-Lik: -2664.901, Max-Change: 0.00024
Iteration: 21, Log-Lik: -2664.901, Max-Change: 0.00020
Iteration: 22, Log-Lik: -2664.901, Max-Change: 0.00018
Iteration: 23, Log-Lik: -2664.901, Max-Change: 0.00016
Iteration: 24, Log-Lik: -2664.901, Max-Change: 0.00014
Iteration: 25, Log-Lik: -2664.901, Max-Change: 0.00014
Iteration: 26, Log-Lik: -2664.901, Max-Change: 0.00011
Iteration: 27, Log-Lik: -2664.901, Max-Change: 0.00009
```

```
coef(m_rasch, IRTpars = TRUE, simplify = TRUE) # d = intercept = -1 * difficulty
```

```
$items
       a      b g u
Item.1 1 -1.868 0 1
Item.2 1 -0.791 0 1
Item.3 1 -1.461 0 1
Item.4 1 -0.521 0 1
Item.5 1 -1.993 0 1

$means
F1
 0

$cov
      F1
F1 1.022
```

The 1PL is just a reparameterization of the Rasch model:

```
# 1PL
mod_1pl <- "
THETA = 1-5
CONSTRAIN = (1-5, a1)
"
m_1pl <- mirt(dat, mod_1pl)
```

```
Iteration: 1, Log-Lik: -2668.786, Max-Change: 0.06198
Iteration: 2, Log-Lik: -2666.196, Max-Change: 0.03935
Iteration: 3, Log-Lik: -2665.347, Max-Change: 0.02405
Iteration: 4, Log-Lik: -2664.955, Max-Change: 0.00867
```

```
Iteration: 5, Log-Lik: -2664.920, Max-Change: 0.00509
Iteration: 6, Log-Lik: -2664.907, Max-Change: 0.00294
Iteration: 7, Log-Lik: -2664.902, Max-Change: 0.00135
Iteration: 8, Log-Lik: -2664.901, Max-Change: 0.00079
Iteration: 9, Log-Lik: -2664.901, Max-Change: 0.00046
Iteration: 10, Log-Lik: -2664.901, Max-Change: 0.00022
Iteration: 11, Log-Lik: -2664.901, Max-Change: 0.00011
Iteration: 12, Log-Lik: -2664.901, Max-Change: 0.00006
```

```
coef(m_1pl, IRTpars = TRUE, simplify = TRUE) # b = difficulty
```

```
$items
           a      b g u
Item.1 1.011 -1.848 0 1
Item.2 1.011 -0.782 0 1
Item.3 1.011 -1.445 0 1
Item.4 1.011 -0.516 0 1
Item.5 1.011 -1.971 0 1

$means
THETA
    0

$cov
      THETA
THETA     1
```

> **Q1**
>
> What is the relationship between (a) the estimated variance of the latent variable (labelled "COV_11") in the 1PL model and (b) the a parameter in the Rasch model?
> Your answer:

Standard Errors

```
m_rasch <- mirt(dat, model = 1, itemtype = "Rasch", SE = TRUE)
```

```
Iteration: 1, Log-Lik: -2664.942, Max-Change: 0.01075
```

```
Iteration: 2, Log-Lik: -2664.913, Max-Change: 0.00257
Iteration: 3, Log-Lik: -2664.910, Max-Change: 0.00210
Iteration: 4, Log-Lik: -2664.908, Max-Change: 0.00179
Iteration: 5, Log-Lik: -2664.906, Max-Change: 0.00156
Iteration: 6, Log-Lik: -2664.905, Max-Change: 0.00137
Iteration: 7, Log-Lik: -2664.904, Max-Change: 0.00141
Iteration: 8, Log-Lik: -2664.903, Max-Change: 0.00110
Iteration: 9, Log-Lik: -2664.903, Max-Change: 0.00095
Iteration: 10, Log-Lik: -2664.902, Max-Change: 0.00086
Iteration: 11, Log-Lik: -2664.902, Max-Change: 0.00072
Iteration: 12, Log-Lik: -2664.902, Max-Change: 0.00063
Iteration: 13, Log-Lik: -2664.902, Max-Change: 0.00065
Iteration: 14, Log-Lik: -2664.901, Max-Change: 0.00051
Iteration: 15, Log-Lik: -2664.901, Max-Change: 0.00044
Iteration: 16, Log-Lik: -2664.901, Max-Change: 0.00040
Iteration: 17, Log-Lik: -2664.901, Max-Change: 0.00034
Iteration: 18, Log-Lik: -2664.901, Max-Change: 0.00029
Iteration: 19, Log-Lik: -2664.901, Max-Change: 0.00030
Iteration: 20, Log-Lik: -2664.901, Max-Change: 0.00024
Iteration: 21, Log-Lik: -2664.901, Max-Change: 0.00020
Iteration: 22, Log-Lik: -2664.901, Max-Change: 0.00018
Iteration: 23, Log-Lik: -2664.901, Max-Change: 0.00016
Iteration: 24, Log-Lik: -2664.901, Max-Change: 0.00014
Iteration: 25, Log-Lik: -2664.901, Max-Change: 0.00014
Iteration: 26, Log-Lik: -2664.901, Max-Change: 0.00011
Iteration: 27, Log-Lik: -2664.901, Max-Change: 0.00009

Calculating information matrix...
```

```r
coef(m_rasch, IRTpars = TRUE)
```

```
$Item.1
         a      b  g  u
par      1 -1.868  0  1
CI_2.5  NA -2.065 NA NA
CI_97.5 NA -1.671 NA NA

$Item.2
         a      b  g  u
par      1 -0.791  0  1
CI_2.5  NA -0.950 NA NA
CI_97.5 NA -0.632 NA NA
```

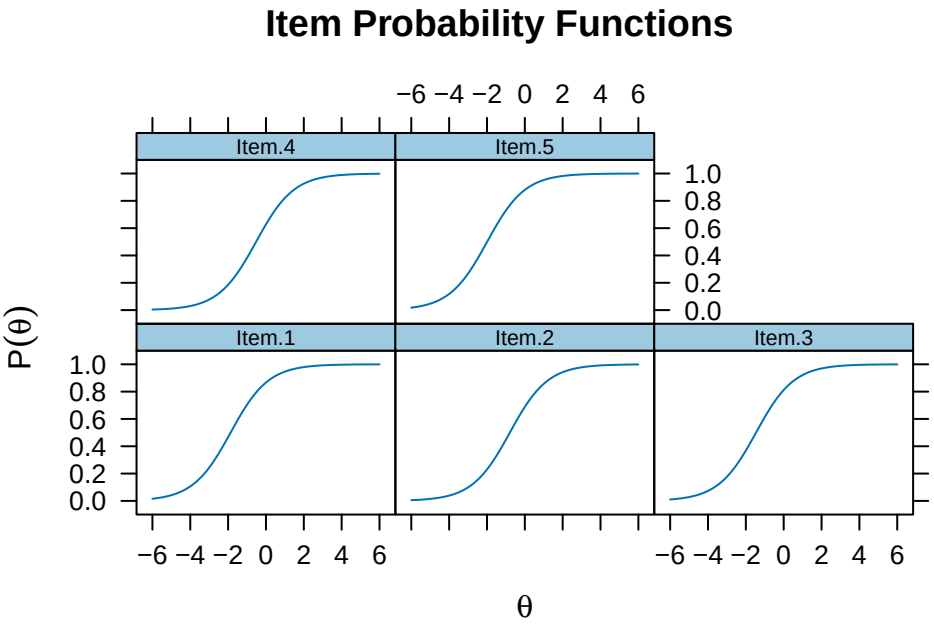
```
$Item.3
         a      b  g  u
par      1 -1.461  0  1
CI_2.5  NA -1.640 NA NA
CI_97.5 NA -1.282 NA NA

$Item.4
         a      b  g  u
par      1 -0.521  0  1
CI_2.5  NA -0.676 NA NA
CI_97.5 NA -0.367 NA NA

$Item.5
         a      b  g  u
par      1 -1.993  0  1
CI_2.5  NA -2.196 NA NA
CI_97.5 NA -1.790 NA NA

$GroupPars
        MEAN_1 COV_11
par          0  1.022
CI_2.5      NA  0.765
CI_97.5     NA  1.279
```

Plots

Item Response Function

Also *item characteristic curve*

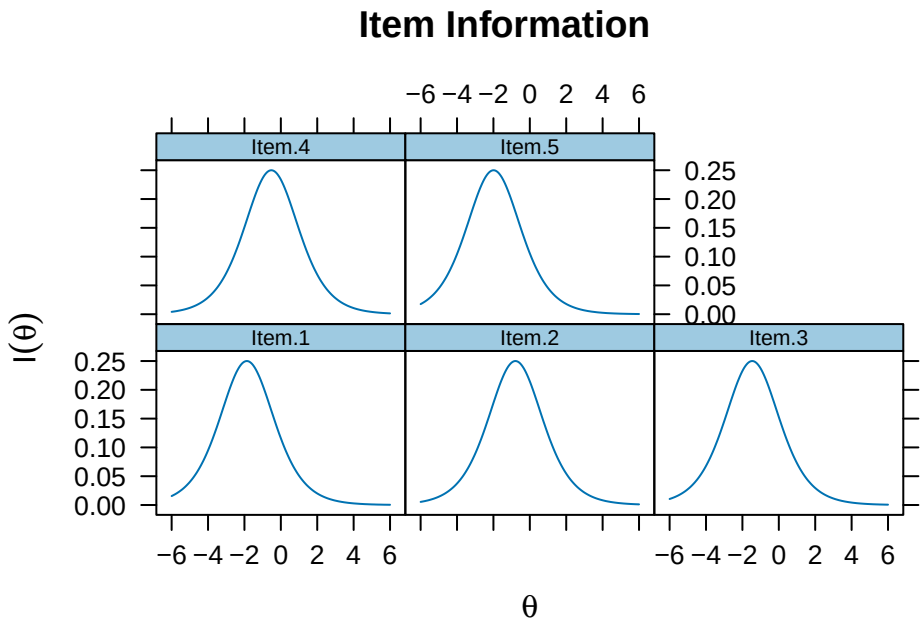
```
plot(m_rasch, type = "trace")
```



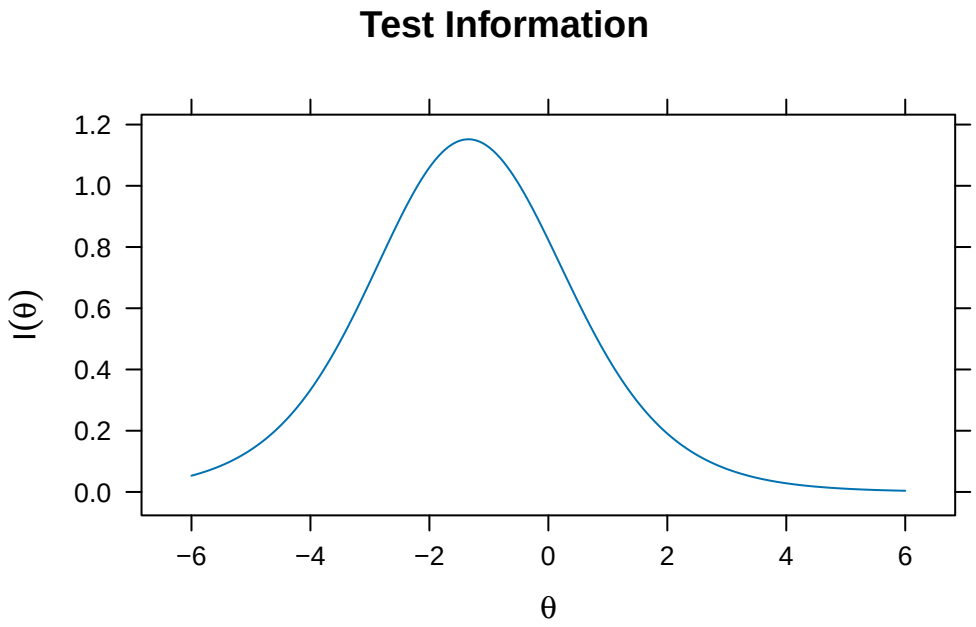
Item and Test Information

Item information

```
plot(m_rasch, type = "infotrace")
```




```
plot(m_rasch, type = "info")
```



Q2

In IRT, the reliability is typically different across different levels of θ. One way to compute reliability that is comparable to the reliability coefficient in CTT is

$$\text{Reliability} = \frac{Var(\theta)}{Var(\theta) + 1/I(\theta)}$$

What level of θ corresponds to the highest reliability in the Rasch model, and what value is the reliability coefficient?
Your answer:

Assumptions/Model Fit

Global test statistics (analogous to those in factor analysis)

- G^2 test statistic (not valid when matrix is sparse)
- M_2: better approximation for polytomous models

```
m_rasch
```

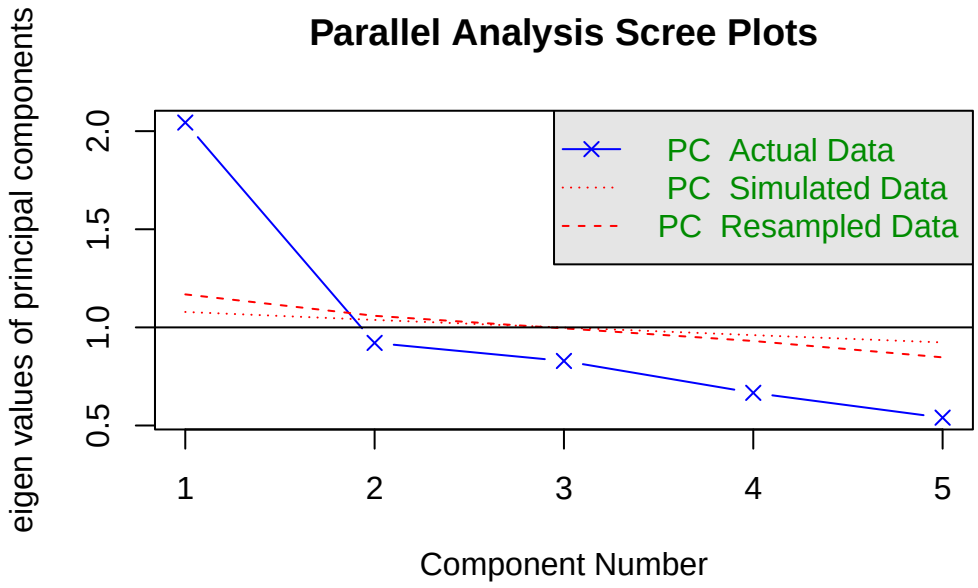
```
Call:
mirt(data = dat, model = 1, itemtype = "Rasch", SE = TRUE)

Full-information item factor analysis with 1 factor(s).
Converged within 1e-04 tolerance after 27 EM iterations.
mirt version: 1.41
M-step optimizer: nlminb
EM acceleration: Ramsay
Number of rectangular quadrature: 61
Latent density type: Gaussian

Information matrix estimated with method: Oakes
Second-order test: model is a possible local maximum
Condition number of information matrix =  4.488772

Log-likelihood = -2664.901
Estimated parameters: 6
AIC = 5341.802
BIC = 5371.248; SABIC = 5352.192
G2 (25) = 43.89, p = 0.0112
RMSEA = 0.028, CFI = NaN, TLI = NaN
```

```
M2(m_rasch)
```

```
            M2 df          p      RMSEA    RMSEA_5   RMSEA_95      SRMSR
stats 23.17287  9 0.00581954 0.03970314 0.02003961 0.05998303 0.04744033
            TLI       CFI
stats 0.9284234 0.9355811
```

Parallel analysis

```
psych::fa.parallel(dat, cor = "poly", fa = "pc")
```




```
Parallel analysis suggests that the number of factors =  NA  and the number of components =
```

Local Independence

```
residuals(m_rasch, type = "Q3")  # Q3
```

```
Q3 summary statistics:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 -0.196  -0.145  -0.116  -0.110  -0.082   0.000

       Item.1 Item.2 Item.3 Item.4 Item.5
Item.1  1.000 -0.136 -0.099 -0.080 -0.051
Item.2 -0.136  1.000  0.000 -0.196 -0.169
Item.3 -0.099  0.000  1.000 -0.133 -0.090
Item.4 -0.080 -0.196 -0.133  1.000 -0.149
Item.5 -0.051 -0.169 -0.090 -0.149  1.000
```

```
# All |Q3| < .2 in this case
```

Item Fit

S-χ^2

```
itemfit(m_rasch)
```

```
    item   S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
1 Item.1  5.593       2      0.042  0.061
2 Item.2 12.188       2      0.071  0.002
3 Item.3  8.589       2      0.057  0.014
4 Item.4  8.987       2      0.059  0.011
5 Item.5  4.877       2      0.038  0.087
```

```
# For large number of items, the p values should be adjusted
itemfit(m_rasch, p.adjust = "holm")
```

```
    item   S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
1 Item.1  5.593       2      0.042  0.122
2 Item.2 12.188       2      0.071  0.011
3 Item.3  8.589       2      0.057  0.045
4 Item.4  8.987       2      0.059  0.045
5 Item.5  4.877       2      0.038  0.122
```

> **Q3**
>
> Which item(s) are not fitting well with the Rasch model?
> Your answer:

Two-Parameter Logistic

```
m_2pl <- mirt(dat, model = 1, itemtype = "2PL")
```

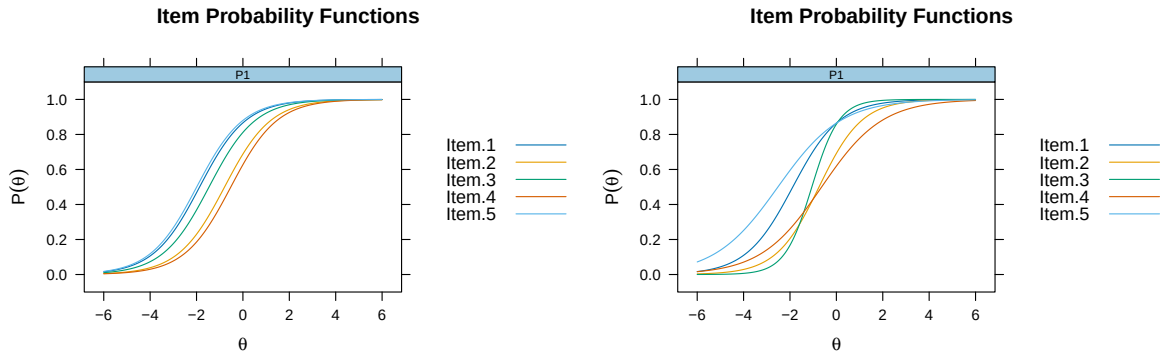
```
Iteration: 1, Log-Lik: -2668.786, Max-Change: 0.18243
Iteration: 2, Log-Lik: -2663.691, Max-Change: 0.13637
Iteration: 3, Log-Lik: -2661.454, Max-Change: 0.10231
Iteration: 4, Log-Lik: -2659.430, Max-Change: 0.04181
Iteration: 5, Log-Lik: -2659.241, Max-Change: 0.03417
Iteration: 6, Log-Lik: -2659.113, Max-Change: 0.02911
Iteration: 7, Log-Lik: -2658.812, Max-Change: 0.00456
Iteration: 8, Log-Lik: -2658.809, Max-Change: 0.00363
Iteration: 9, Log-Lik: -2658.808, Max-Change: 0.00273
Iteration: 10, Log-Lik: -2658.806, Max-Change: 0.00144
Iteration: 11, Log-Lik: -2658.806, Max-Change: 0.00118
Iteration: 12, Log-Lik: -2658.806, Max-Change: 0.00101
Iteration: 13, Log-Lik: -2658.805, Max-Change: 0.00042
Iteration: 14, Log-Lik: -2658.805, Max-Change: 0.00025
Iteration: 15, Log-Lik: -2658.805, Max-Change: 0.00026
Iteration: 16, Log-Lik: -2658.805, Max-Change: 0.00023
Iteration: 17, Log-Lik: -2658.805, Max-Change: 0.00023
Iteration: 18, Log-Lik: -2658.805, Max-Change: 0.00021
Iteration: 19, Log-Lik: -2658.805, Max-Change: 0.00019
Iteration: 20, Log-Lik: -2658.805, Max-Change: 0.00017
Iteration: 21, Log-Lik: -2658.805, Max-Change: 0.00017
Iteration: 22, Log-Lik: -2658.805, Max-Change: 0.00015
Iteration: 23, Log-Lik: -2658.805, Max-Change: 0.00015
Iteration: 24, Log-Lik: -2658.805, Max-Change: 0.00013
Iteration: 25, Log-Lik: -2658.805, Max-Change: 0.00013
Iteration: 26, Log-Lik: -2658.805, Max-Change: 0.00011
Iteration: 27, Log-Lik: -2658.805, Max-Change: 0.00011
Iteration: 28, Log-Lik: -2658.805, Max-Change: 0.00010
```

```
coef(m_2pl, IRTpars = TRUE, simplify = TRUE)
```

```
$items
           a      b g u
Item.1 0.988 -1.879 0 1
Item.2 1.081 -0.748 0 1
```

```
Item.3 1.706 -1.058 0 1
Item.4 0.765 -0.635 0 1
Item.5 0.736 -2.520 0 1

$means
F1
 0

$cov
   F1
F1  1
```

```r
plot(m_rasch, type = "trace", facet_items = FALSE)
plot(m_2pl, type = "trace", facet_items = FALSE)
```



(a) Rasch model

(b) 2PL model

Figure 1: Item response functions, aka item characteristic curves

Q4

Plot the item information curve for the 2-PL model. Which item has the highest information at $\theta = 0$?
Your answer:

```r
# Insert R code
```

Model Comparison

```r
anova(m_rasch, m_2pl)
```

```
            AIC    SABIC       HQ      BIC    logLik     X2 df     p
m_rasch 5341.802 5352.192 5352.994 5371.248 -2664.901
m_2pl   5337.610 5354.927 5356.263 5386.688 -2658.805 12.192  4 0.016
```

Some Polytomous Models

Rating scale model

```r
data(bfi, package = "psych")
# Neuroticism
m_rsm <- mirt(bfi[16:20], itemtype = "rsm")
```

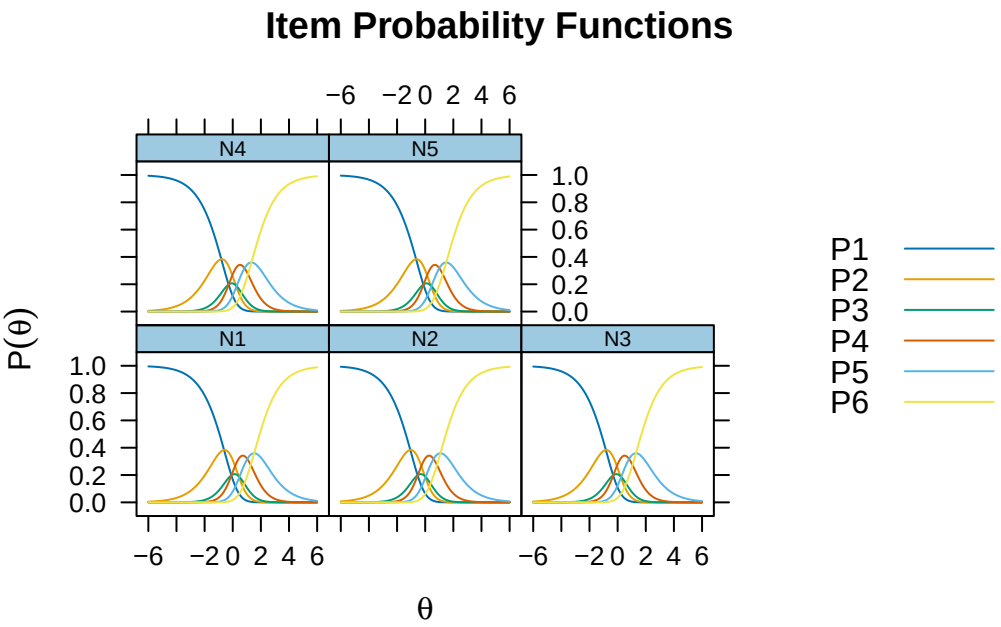
```
Iteration: 1, Log-Lik: -23985.274, Max-Change: 1.00910
Iteration: 2, Log-Lik: -22434.284, Max-Change: 0.18792
Iteration: 3, Log-Lik: -22338.797, Max-Change: 0.17826
Iteration: 4, Log-Lik: -22282.201, Max-Change: 0.12715
Iteration: 5, Log-Lik: -22244.160, Max-Change: 0.09083
Iteration: 6, Log-Lik: -22217.562, Max-Change: 0.06646
Iteration: 7, Log-Lik: -22198.665, Max-Change: 0.04953
Iteration: 8, Log-Lik: -22185.225, Max-Change: 0.03738
Iteration: 9, Log-Lik: -22175.730, Max-Change: 0.02839
Iteration: 10, Log-Lik: -22169.095, Max-Change: 0.02270
Iteration: 11, Log-Lik: -22164.507, Max-Change: 0.01858
Iteration: 12, Log-Lik: -22161.370, Max-Change: 0.01513
Iteration: 13, Log-Lik: -22156.185, Max-Change: 0.01992
Iteration: 14, Log-Lik: -22155.234, Max-Change: 0.00485
Iteration: 15, Log-Lik: -22155.115, Max-Change: 0.00311
Iteration: 16, Log-Lik: -22154.996, Max-Change: 0.00344
Iteration: 17, Log-Lik: -22154.963, Max-Change: 0.00141
Iteration: 18, Log-Lik: -22154.950, Max-Change: 0.00099
Iteration: 19, Log-Lik: -22154.935, Max-Change: 0.00132
Iteration: 20, Log-Lik: -22154.930, Max-Change: 0.00040
Iteration: 21, Log-Lik: -22154.929, Max-Change: 0.00027
Iteration: 22, Log-Lik: -22154.928, Max-Change: 0.00033
Iteration: 23, Log-Lik: -22154.928, Max-Change: 0.00012
Iteration: 24, Log-Lik: -22154.928, Max-Change: 0.00008
```

```r
itemfit(m_rsm, na.rm = TRUE)  # not fitting well
```

```
Sample size after row-wise response data removal: 2694
```

```
  item    S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
1   N1 146.044      64      0.022      0
2   N2 141.410      63      0.021      0
3   N3 143.549      63      0.022      0
4   N4 227.688      63      0.031      0
5   N5 320.471      63      0.039      0
```

```r
plot(m_rsm, type = "trace")
```



Graded Response model

```r
m_grm <- mirt(bfi[16:20], itemtype = "graded")
```

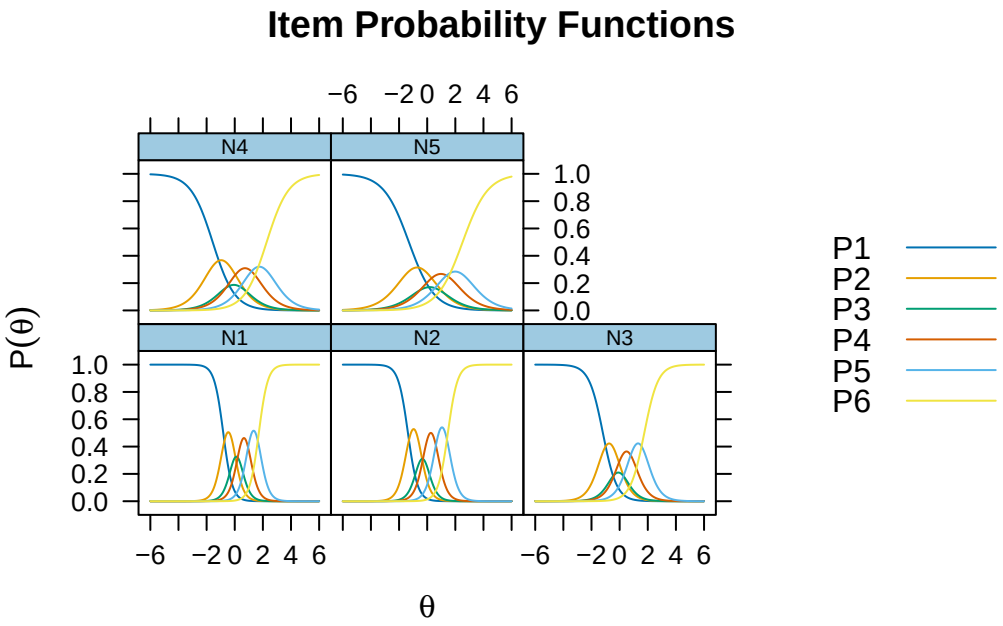
```
Iteration: 1, Log-Lik: -22678.849, Max-Change: 0.98180
Iteration: 2, Log-Lik: -22009.516, Max-Change: 0.62057
Iteration: 3, Log-Lik: -21843.393, Max-Change: 0.31998
```

```
Iteration: 4, Log-Lik: -21773.416, Max-Change: 0.25006
Iteration: 5, Log-Lik: -21747.311, Max-Change: 0.16264
Iteration: 6, Log-Lik: -21734.951, Max-Change: 0.15148
Iteration: 7, Log-Lik: -21728.377, Max-Change: 0.09627
Iteration: 8, Log-Lik: -21724.841, Max-Change: 0.07974
Iteration: 9, Log-Lik: -21722.917, Max-Change: 0.06180
Iteration: 10, Log-Lik: -21721.933, Max-Change: 0.02070
Iteration: 11, Log-Lik: -21721.615, Max-Change: 0.01547
Iteration: 12, Log-Lik: -21721.511, Max-Change: 0.01199
Iteration: 13, Log-Lik: -21721.392, Max-Change: 0.00246
Iteration: 14, Log-Lik: -21721.388, Max-Change: 0.00214
Iteration: 15, Log-Lik: -21721.385, Max-Change: 0.00159
Iteration: 16, Log-Lik: -21721.382, Max-Change: 0.00202
Iteration: 17, Log-Lik: -21721.381, Max-Change: 0.00050
Iteration: 18, Log-Lik: -21721.381, Max-Change: 0.00008
```

```r
itemfit(m_grm, na.rm = TRUE)  # not fitting well
```

```
Sample size after row-wise response data removal: 2694

  item    S_X2 df.S_X2 RMSEA.S_X2 p.S_X2
1   N1  99.122      56      0.017  0.000
2   N2 142.420      55      0.024  0.000
3   N3  87.896      63      0.012  0.021
4   N4 105.378      70      0.014  0.004
5   N5 115.295      72      0.015  0.001
```

```r
plot(m_grm, type = "trace")
```




```
plot(m_grm, type = "info")
```